A STUDY ON MATHEMATICAL MODELLING IN FLUID FLOW APPLICATIONS USING PARTIAL DIFFERENTIAL EQUATIONS

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Abstract

The essential feature of the Functioning of Derivates approach is the structuring of optimization problems into multiple stages, which are solved sequentially one stage at a time. Although each one-stage problem is solved as an ordinary optimization problem, its solution helps to define the characteristics of the next one-stage problem in the sequence. Often, the stages represent different time periods in the problem's planning horizon. For example, the problem of determining the level of inventory of a single commodity can be stated as a derivate program. There are many cases in practical applications where the variables of optimization are not continuous. Some or all of the variables must be selected from a list of integer or discrete values.

Keywords: derivates, mathematical functions

INTRODUCTION

The famous naturalist Charles Darwin defined natural selection or survival of the fittest in his book (Darwin, 1929) as the preservation of favourable individual differences and variations, and the destruction of those that are injurious. In nature, individuals have to adapt to their environment in order to survive in a process called evolution, in which those features that make an individual more suitable to compete are preserved when it reproduces, and those features that make it weaker are eliminated. Such features are controlled by units called genes, which form sets known as chromosomes. Over subsequent generations not only the fittest individuals survive, but also their genes which are transmitted to their descendants during the sexual recombination process, which is called crossover.

A review of the methods for discrete variable optimization was recently presented by Bremicker et al. (1990), Vanderplaats and Thanedar (1991) and Arora et al. (1994). Several algorithms for discrete optimization problems were developed, among them branch and bound method, penalty function approach, rounding–off, cutting plane, simulated annealing, genetic algorithms, neural networks, and Lagrangian relaxation methods. It is observed that some of the methods for discrete variable optimization use the structure of the problem to speed up the search for the discrete solution. This class of methods is not suitable for implementation into a general purpose application (Arora et al., 1994). The branch and bound

method, simulated annealing, and genetic algorithm are the most used methods. Herein, the literature review will be focused on these methods in the following sections.

BACKGROUND OF THE STUDY

This paper, published in two parts, is mainly concerned with general properties of Dini derivatives of functions of one and several variables and with some applications of this topic to the study of generalized convexity and generalized optimality conditions for mathematical programming problems. In part I the basic definitions and properties are given, with reference both to functions of one real variable and to functions of several real variables. In this part special attention is given to the restatement of the basic theorems of the classical analysis to non-differentiable functions, in terms of Dini derivatives. In part II we use these derivatives in order to define some classes of no differentiable generalized convex functions and the class of generalized upper quasi differentiable functions. This part concludes with the development of optimality conditions for a non-smooth programming problem, expressed in terms of the tools previously introduced.

SCOPE OF WORK

This article aims to provide a process that can be used in financial risk management by resolving problems of minimizing the risk measure (VaR) using derivatives products, bonds and options. This optimization problem was formulated in the hedging situation of a portfolio formed by an active and a put option on this active, respectively a bond and an option on this bond. In the first optimization problem we will obtain the coverage ratio of the optimal price for the excretion of the option which is in fact the relative cost of the option's value. In the second optimization problem we obtained optimal exercise price for a put option which is to support a bond.

OBJECTIVES

1. To check the developments in derivatives of mathematics with genetic algorithm.
2. To find out the scope and need of derivatives in mathematics with applications.
3. To find the Applications of the Derivative.
4. To check the Applications of Derivatives to Business and Economics.
5. To check A Collection of Problems in Differential Calculus.
6. To find An Optimization of the Risk Management using Derivatives.

LITERTAURE SURVEY

Hobden P (2006) this article aims to provide a process that can be used in financial risk management by resolving problems of minimizing the risk measure (VaR) using derivatives products, bonds and options. This optimization problem was formulated in the hedging situation of a portfolio formed by an active and a put option on this active, respectively a bond and an option on this bond. In the first optimization problem we will obtain the coverage ratio of the optimal price for the excertion of the option which is in fact the relative cost of the option's value. In the second optimization problem we obtained optimal exercise price for a put option which is to support a bond.

Maharajh N, Brijlall D & Govender N (2008) In the article's first part we obtained the coverage ratio of the optimal exercise price of options which is the relative cost at the option's value by an optimization problem of VaR in the situation of a portfolio consisting in a financial asset and a put option. In the second part of this article we obtained the optimal exertion price of a put option in a portfolio in which is a bond and this option. And for this result it has been created a minimum problem of VaR and the obtained result leads to the idea that VaR linear depends of the h percent of the bond cover by options.

Marrongele K(2010) Many important applied problems involve finding the best way to accomplish some task. Often this involves finding the maximum or minimum value of some function: the minimum time to make a certain journey, the minimum cost for doing a task, the maximum power that can be generated by a device, and so on. Many of these problems can be solved by finding the appropriate function and then using techniques of calculus to find the maximum or the minimum value required.

Bezuidenhout J (2001) This qualitative case study in a rural school in Umgungundlovu District in KwaZulu-Natal, South Africa, explored Grade 12 learners' mental constructions of mathematical knowledge during engagement with optimisation problems. Ten Grade 12 learners who do pure Mathematics participated, and data were collected through structured activity sheets and semi-structured interviews. Structured activity sheets with three tasks were given to learners; these tasks were done in groups, and the group leaders were interviewed. It was found that learners tended to do well with routine-type questions, implying that they were functioning at an action level. From the interviews it appeared that learners might have the correct answer, but lacked conceptual understanding.

METHODOLOGY

These methods approximate an unknown function f by a response surface (or meta model)ˆf. Any mismatch between f and ˆf is assumed to be caused by model error and not because of noise in experimental measurements. Response surfaces may be non-interpolating or interpolating. The former are obtained by minimizing the sum of square deviations between f and ˆfat a number of points, where measurements of f have been obtained. The latter produce functions that pass through the sampled responses. x

$$\hat{f}(x) = \sum_{i=1}^{m} \alpha_i f_i(x) + \sum_{i=1}^{p} \beta_i \varphi\left(x - x^{(i)}\right)$$

Where fi are polynomial functions, αi and βi are unknown coefficients to be estimated, ϕ is a basis function, and x(i)∈Rn, i=1,..., p, are sample points. Basis functions include linear, cubic, thin plate splines, multi quadratic, and kriging. These are discussed below in more detail. Kriging Originally used for mining exploration models, kriging models a deterministic response as the realization of a stochastic process by means of a kriging basis function.

$$\hat{f}(x) = \mu + \sum_{i=1}^{p} b_i \exp\left[-\sum_{h=1}^{n} \theta_h \left|x_h - x_h^{(i)}\right|^{p_h}\right],$$
$$\theta_h \geq 0, \quad p_h \in [0, 2], h = 1, \ldots, n.$$

Assuming ˆf is a random variable with known realizations ˆf(x(i)),i=1,..., p, the parameters μ, b i, θh and ph are estimated by maximizing the likelihood of the observed realizations. The parameters are dependent on sample point information but independent of the candidate point x. Nearby points are assumed to have highly correlated function values, thus generating a continuous interpolating model. The weights θh and ph account for the importance and the smoothness of the corresponding variables. The predictor ˆf(x) is the minimized over the entire domain.

RESULTS

In order to assess the quality of the solutions obtained by different solvers, we compared the solutions returned by the solvers against the globally optimal solution for each problem. A solver was considered to have successfully solved a problem during a run if it returned a solution with an objective function value within 1% or 0.01 of the global optimum, which ever was larger. In other words, a solver was considered successful if it reported a solution y such that $f(y) \leq \max(1.01 f(x*), f(x*)+0.01)$, where x*is a globally optimal solution for the problem. To obtain global optimal solutions for the test problems, we used the general-purpose global optimization solver BARON to solve as many of the test problems as possible. Unlike derivative-free solvers, BARON requires explicit algebraic expressions rather than function values alone. BARON's branch-and-bound strategy was able to guarantee global optimality for most of the test problems, although this solver does not accept trigonometric and some other nonlinear functions. For the latter problems, LINDOGLOBAL was used to obtain a global solution.
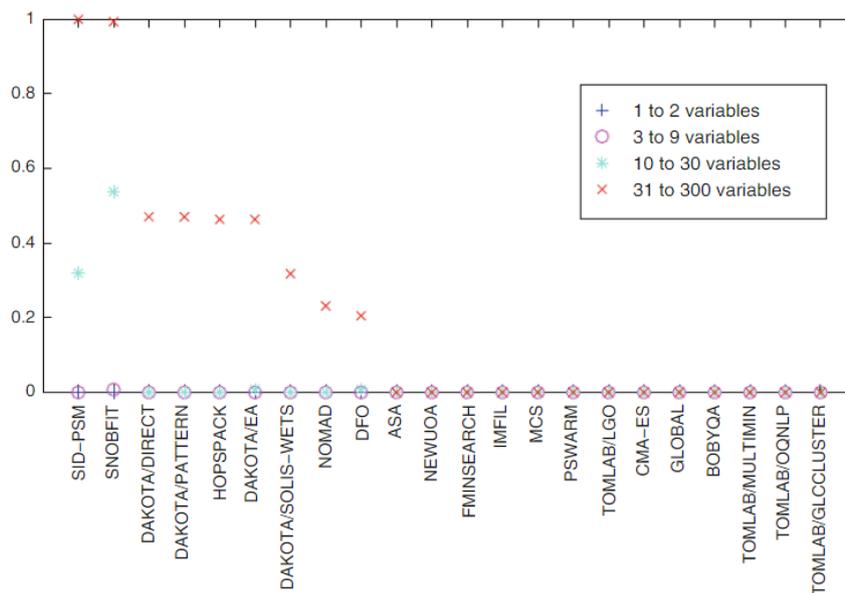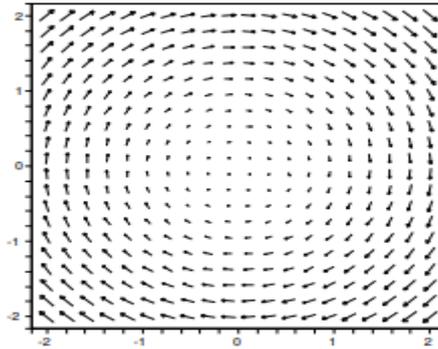
Examples of divergence free vector fields in R^2

Most instances were solved within a few minutes. Since the test problems are algebraically and computationally simple and small, the total time required for function evaluations for all runs was negligible. Most of the CPU time was spent by the solvers on processing function values and determining the sequence of iterates. A limit of 10 CPU minutes was imposed on each run.

A high-temperature solid metal cylinder of radius mr and length L is cooled with a water flow moving longitudinally in the direction of the axis ɯ and having the initial rate 0 u and temperature T0l . The water thickness is l m r_l-r_m. Above the water layer there is a layer of air with thickness al r_l-r_m . The physical diagram of the calculation region is presented
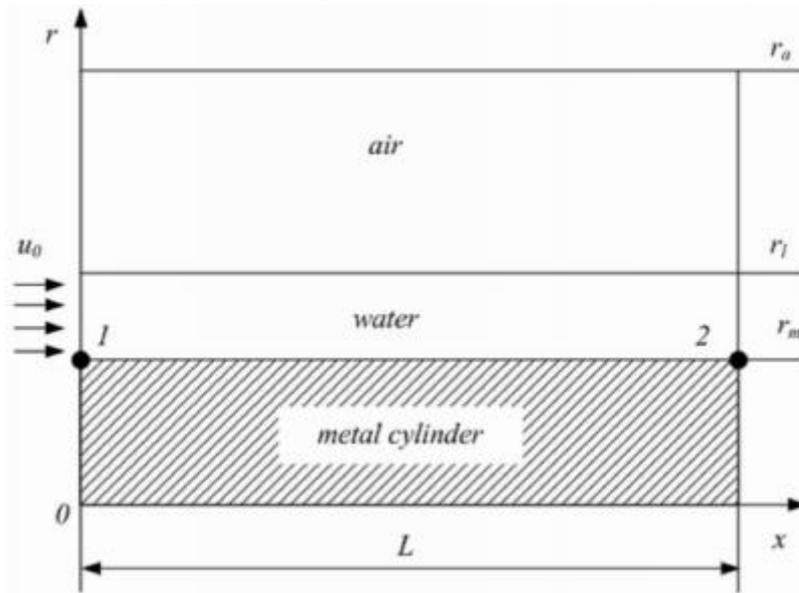
The physical diagram of the calculation region

The system of equations for the gas-liquid medium

R_m<R<R_a

$$\bar{\rho}\frac{\partial u}{\partial t}+\bar{\rho}u\frac{\partial u}{\partial x}+\bar{\rho}v\frac{\partial u}{\partial r}=-\frac{\partial p}{\partial x}+\frac{\partial}{\partial x}\bar{\mu}\frac{\partial u}{\partial x}+\frac{1}{r}\frac{\partial}{\partial r}r\bar{\mu}\frac{\partial u}{\partial r}$$

.... (1)

$$\bar{\rho}\frac{\partial v}{\partial t}+\bar{\rho}u\frac{\partial v}{\partial x}+\bar{\rho}v\frac{\partial v}{\partial r}=-\frac{\partial p}{\partial r}+\frac{\partial}{\partial x}\bar{\mu}\frac{\partial v}{\partial x}+\frac{1}{r}\frac{\partial}{\partial r}r\bar{\mu}\frac{\partial v}{\partial r}-\bar{\mu}\frac{v}{r^2}$$

.... (2)

$$\frac{\partial \bar{\rho}}{\partial t} + \frac{\partial(\bar{\rho}u)}{\partial x} + \frac{1}{r}\frac{\partial(r\bar{\rho}v)}{\partial r} = 0$$

.... (3)

$$\bar{\rho c}\frac{\partial T}{\partial t} + \bar{\rho c}u\frac{\partial(\bar{\rho}u)}{\partial x} + \bar{\rho c}v\frac{\partial T}{\partial r} = \frac{\partial}{\partial x}\bar{\lambda}\frac{\partial T}{\partial x} + \frac{1}{r}\frac{\partial}{\partial r}r\bar{\lambda}\frac{\partial T}{\partial r} - \dot{m}Q$$

.... (4)

$$\bar{\rho}\frac{\partial Y}{\partial t} + \bar{\rho}u\frac{\partial Y}{\partial x} + \bar{\rho}v\frac{\partial Y}{\partial r} = \dot{m}.$$

.... (5)

The specific mass rate of the vapor formation is found from the heat balance equation

$$\dot{m} = \left(\overline{\rho c}\,\Delta T^*\right)\Big/ Q,$$

where the reduced heat flow is defined according to the following equation

$$\Delta T^* = \begin{cases} 0, & \text{if } T(t+\Delta t) < T_S \\ \left[T(t+\Delta t) - T(t)\right]\big/\Delta t, & \text{if } T(t+\Delta t) > T_S \end{cases}$$

The energy equation for the metal cylinder
$0 < r < r_m$

Calculation results:

The results obtained with the use of the offered mathematical model allowing solving the conjugate problem of convective heat exchange taking into account the vapor formation with the results calculated by the mathematical model using criterial equations for determining the boundary conditions on the cylinder surface at the contact with the water flow. Let us consider cooling a solid metal cylinder borrowing the initial data from $R_m = 0.01$ m3, $R_l = 0.013$m, L 0.2 m, 0 20 T l °C. Let us take 0,1 ar m, the air initial temperature T T 0 0 a l, the initial temperature of the cylinder 950 Tm °C, the saturation temperature 100 Ts °C, and the water flow rate $u0 = 12$ m/s. The calculation time is 10 s. The thermophysical parameters of the media are taken according to
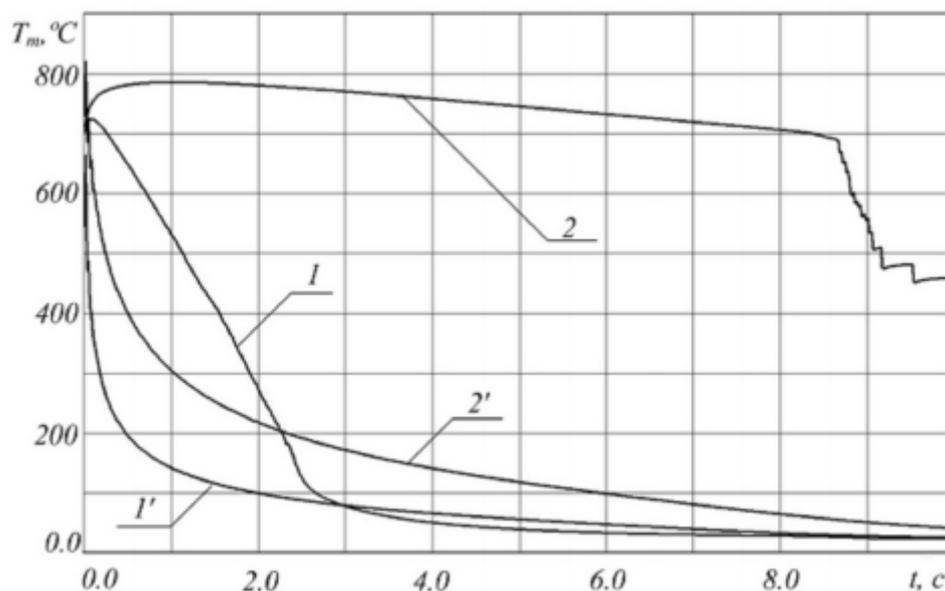
The temperature of the cylinder surface at the calculation points:1, 2 – the temperatures calculated with the use of the offered model; 1', 2' – the temperatures calculated using the model described
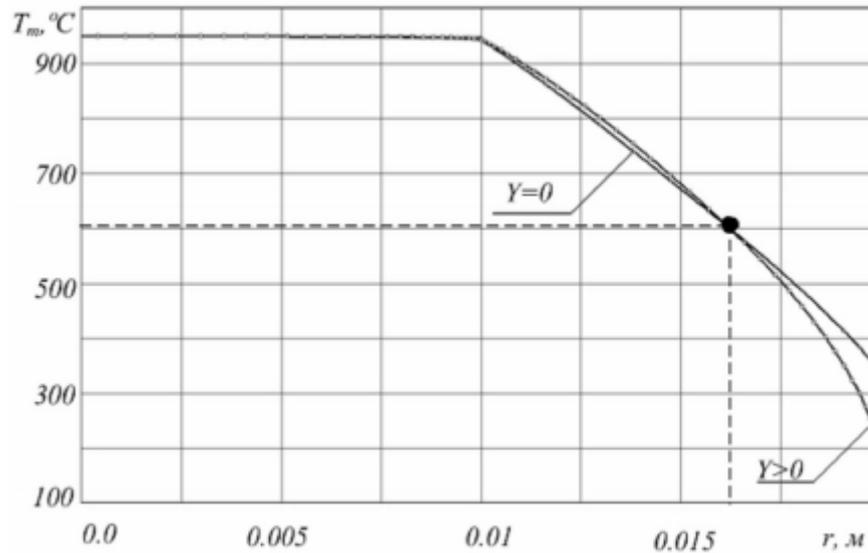
The temperature in the middle of the cylinder surface when the presence of vapor Y>0 is taken into account, and when the presence of vapor Y=0 is not taken into account

When all the other conditions are equal, the higher the water flow rate is, the lower the received values of the temperature at the calculation points are. This regularity is more obvious in the initial region of the cooled surface. At the calculation point 1 within the time 5 s, the difference in the temperatures makes up 45% at the rates 1 m/s and 10 m/s. At the calculation point 2, when the rate of the flow decreases from 10 m/s to 1 m/s, the temperature increases by 4%, i.e. it remains practically the same for all the considered values of the water flow rate

Most instances were solved within a few minutes. Since the test problems are algebraically and computationally simple and small, the total time required for function evaluations for all runs was negligible. A limit of 10 CPU minutes was imposed on each run. As seen in this figure, no solver reached this CPU time limit for problems with up to nine variables. For problems with ten to thirty variables, only SID-PSM and SNOBFIT had to be terminated because of the time limit.

CONCLUSION

Although this was a small-scale study and the findings cannot be generalised, the formulation of IGD and the theory of APOS provided a valuable way of exploring the learning of maximising/minimising in Calculus in a Grade 12 Mathematics class. The study revealed that some aspects of APOS theory were not fully operational, because most of the learners were successful with the questions where they could use formulas and substitute when necessary. These manipulations took place externally, as they could follow the given instruction and apply rules that they had memorised. Some of them had acquired the process stage of APOS theory, like Group 3, who interiorised the volume formula into a process where they showed understanding that for maxima/minima to occur, the gradient at that point is zero. Among the

learners, L2 seemed to have the maxima/minima schema developed. Based on the genetic decomposition presented at the beginning of each analysis, the study showed that learners were partially in line with the IGD, but the schema part was lacking in their attempt to construct knowledge of the learnt concept. The study showed that learners" construction of knowledge was mainly based upon isolated facts and procedures. This might be a result of the way in which teaching and learning occurred, which put more emphasis on procedural aspects in Calculus, neglecting conceptual understanding of the concept. The exercises that learners were doing. the results of the mathematical modeling can be used for solving mathematical problems of finding initial heat and hydrodynamic conditions for the calculation of heat-stressed processes in metallurgy and mechanical engineering, for example, for the calculation of the parameters of cooling high-temperature cylindrical metal work pieces depending on geometry and thermophysical properties of cooled material and cooling media, and the time of the process.

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