Effects of Laminar Skin Friction and Free Convection on MHD Boundary Layer Flow

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Abstract:

Free convection is evident with a large number of analytical, numerical and experimental works considering various problems in pure free convection and combined free and forced convection both in internal flows and in boundary layer flows. Our present study is based mainly on the analysis made by Soundalgekar, in which an extensive theoretical study of free convection effects on oscillatory flow has been presented. The mean and fluctuating flow in two separate parts emphasizing the former one have been discussed. Here, in our study we have presented the magnetic aspect of the above problem with the consideration of Joulean dissipation term in the energy equation and changed boundary condition at infinity. Also, we have assumed variable suction velocity at the plate. It has been concluded by Soundalgekar that owing to the greater viscous dissipative heat, the mean skin friction always increases. Our investigation leads to the conclusion that in the presence of magnetic field, greater viscous dissipative heat leads to a decrease in the mean skin friction of air and an increase in the mean skin friction of water.

Introduction:

Recently, in a series of papers, published by Soundalgekar and his co-workers [6 – 10], an elegant analysis of free convection oscillatory flow has been presented. Soundalgekar and Pop [8], the effects of variable suction velocity on free convection oscillatory flow have been discussed. Soundalgekar [7] studied the free convection effects on the oscillatory flow past an infinite, vertical, porous plate with constant suction and Joulean dissipation terms in the energy equation and solved the coupled non-linear equations by Fourier series method.

Further, the values of mean skin friction and mean heat transfer rate with respect to the parameter G, E and P are entered in the Tables 1 and 2. It has been observed from the Table –1 that due to greater cooling of the plate by free convection currents, mean skin friction increases always. From Table 2 we observe that greater cooling of the plate leads to an increase in the mean heat transfer rate of air and a decrease in the mean heat transfer rate of water. The effects of suction parameter on the amplitudes and phases of the skin friction and heat transfer rate have been discussed. Our analysis is based on Acharya *et al*. [1], Bujurke *et al*. [2], Chauhan and Sahai [3], Gordon *et al*. [4], Lightill [5], Varshney and Ram Prakash [11], Varshney and Sharma [12] and Whittaker and Lister [13]. Such type of theoretical study helps in providing a qualitative insight to the generation of boundary plumes. It has been suggested that in a region of strong magnetic field a boundary line plume is generated which is needed in nuclear fusion reactor blankets.

Mathematical Analysis:

Two dimensional unsteady MHD boundary layer flow of electrically conducting, incompressible, viscous fluid past an infinite vertical porous plate with variable suction is considered. The $\bar{x}$ and $\bar{y}$ axes are taken along and perpendicular to the plate, respectively. A uniformly distributed constant magnetic field B_0 is assumed to act in $\bar{y}$ direction. The free convection boundary layer flow is maintained by the differences between the plate and stream temperatures. Also the flow is assumed to be at small magnetic Reynolds number so that the induced effect can be neglected. With viscous and Joulean dissipation terms taking into consideration in energy equation, the flow under the Boussinesq approximation is governed by :

$$\frac{\partial \bar{v}}{\partial \bar{y}} = 0, \tag{1.1}$$

$$\frac{\partial \bar{u}}{\partial \bar{t}} + \bar{v}\frac{\partial \bar{u}}{\partial \bar{y}} = g_x \beta\left(\bar{T} - \bar{T}_\infty\right) + \nu \frac{\partial^2 \bar{u}}{\partial \bar{y}^2} - \frac{\sigma B_0^{\,2}}{\bar{\rho}}\bar{u}, \tag{1.2}$$

$$\bar{\rho}\, c_p \left(\frac{\partial \bar{T}}{\partial \bar{t}} + \bar{v}\frac{\partial \bar{T}}{\partial \bar{y}}\right) = k\frac{\partial^2 \bar{T}}{\partial \bar{y}^2} + \mu\left(\frac{\partial \bar{u}}{\partial \bar{y}}\right)^2 + \sigma B_0^{\,2}\,\bar{u}^2 . \tag{1.3}$$

The boundary conditions are

$$\left.\begin{aligned} \bar{y} = 0; \quad \bar{u} = 0, \quad \bar{T} = \bar{T}_w, \\ \bar{y} \to \infty; \quad \bar{u} = 0, \quad \bar{T} = \bar{T}_\infty \end{aligned}\right\} \tag{1.4}$$

As we have proposed to study variable suction case, integrating equation (1.1) we get

$$\bar{v} = -v_0 \left(1 + \in A \ e^{i \bar{\omega} \bar{t}}\right), \tag{1.5}$$

where the notation have their usual meaning.

On substitution equation (1.5), the equations (1.2) by virtue of the non-dimensional transformations reduce to

$$\frac{\partial^2 u}{\partial y^2} + \left(1 + \in A \ e^{i \omega t}\right) \frac{\partial u}{\partial y} + G\theta - \frac{1}{4}\frac{\partial u}{\partial t} - \frac{1}{4} M u = 0, \tag{1.6}$$

$$\frac{\partial^2 \theta}{\partial y^2} + P\left(1 + \in A \ e^{i \omega t}\right) \frac{\partial \theta}{\partial y} - \frac{P}{4}\frac{\partial \theta}{\partial t} + P E\left(\frac{\partial u}{\partial y}\right)^2$$

$$+ \frac{1}{4} M E P \ u^2 = 0. \tag{1.7}$$

The non-dimensional boundary conditions are

$$\left.\begin{aligned} y = 0; \quad & u = 0, \quad \theta = 1, \\ y \to \infty; \quad & u = 0, \quad \theta = 0. \end{aligned}\right\} \tag{1.8}$$

Solution by Perturbation Method –

To solve the equations (1.6) and (1.7), we assume

$$\left.\begin{aligned} u(y, t) = u_0(y) + \in e^{i \omega t} \ u_1(y), \end{aligned}\right\} \tag{1.9}$$

$$\theta(y, t) = \theta_0(y) + \in e^{i\omega t}\,\theta_1(y)\,.$$

Substituting equation (1.9) into (1.6) and (1.7), equating the coefficient of harmonic and non-harmonic terms, neglecting the coefficient of $\in^2$, we get

$$u_0'' + u_0' - \frac{1}{4} M\, u_0 = -G\,\theta_0\,, \tag{1.10}$$

$$u_1'' + u_1' - \frac{1}{4}(M + i\omega)\, u_1 = -A\, u_0' - G\,\theta_1\,, \tag{1.11}$$

$$\theta_0'' + P\,\theta_0' = -P\,E\,u_0'^{\,2} - \frac{M E P}{4}\, u_0^{\,2}\,, \tag{1.12}$$

$$\theta_1'' + P\,\theta_1' - \frac{1}{4} i\,\omega\, P\,\theta_1 = -P\,A\,\theta_0' - 2PE\,u_0'\,u_1' - \frac{MEP}{2}\, u_0\, u_1\,. \tag{1.13}$$

In view of equation (1.9), boundary conditions (1.8) reduce to

$$\left.\begin{array}{l} y = 0; \quad u_0 = 0, \quad u_1 = 0; \quad \theta_0 = 1, \quad \theta_1 = 0, \\ y \to \infty; \quad u_0 = 0, \quad u_1 = 0; \quad \theta_0 = 0, \quad \theta_1 = 0 \end{array}\right\} \tag{1.14}$$

In equation (1.10) – (1.13), the primes denote the differentiation with respect to y. These equations are still coupled and non-linear and so are difficult to solve. To solve them, we again expand u_0, u_1, θ_0 and θ_1 in powers of E. Hence we assume

$$u_0(y) = u_{00}(y) + E\ u_{01}(y) + 0(E^2), \tag{1.15}$$

$$u_1(y) = u_{10}(y) + E\, u_{11}(y) + 0(E^2), \tag{1.16}$$

$$\theta_0(y) = \theta_{00}(y) + E\ \theta_{01}(y) + 0(E^2), \tag{1.17}$$

$$\theta_1(y) = \theta_{10}(y) + E\ \theta_{11}(y) + 0(E^2)\ . \tag{1.18}$$

Substituting equations (1.15) – (1.18) in equations (1.10) – (1.13), equating the coefficients of different powers of E, neglecting the coefficient of E^2 and so on, we have

$$u''_{00} + u'_{00} - \frac{M}{4} u_{00} = -G\ \theta_{00}\ , \tag{1.19}$$

$$u''_{01} + u'_{01} - \frac{M}{4} u_{01} = -G\ \theta_{01}\ , \tag{1.20}$$

$$u''_{10} + u'_{10} - \frac{1}{4}(M + i\,\omega) u_{10} = -A\ u'_{00} - G\ \theta_{10}\ , \tag{1.21}$$

$$u''_{11} + u'_{11} - \frac{1}{4}(M + i\,\omega) u_{11} = -A\ u'_{01} - G\ \theta_{11}\ , \tag{1.22}$$

$$\theta''_{00} + P\ \theta'_{00} = 0\ , \tag{1.23}$$

$$\theta''_{01} + P\ \theta'_{01} = -P\ {u'_{00}}^2 - \frac{M P}{4} {u_{00}}^2\ , \tag{1.24}$$

$$\theta''_{10} + P\,\theta'_{10} - \frac{i\,\omega P}{4} \theta_{10} = -P\ A\ \theta'_{00}\ , \tag{1.25}$$

$$\theta_{11}'' + P\,\theta_{11}' - \frac{i\omega P}{4}\,\theta_{11} = -P\,A\,\theta_{01}'$$

$$- 2\,P\,u_{00}'\,u_{10}' - \frac{M P}{2} u_{00}\,u_{10}\,, \qquad (1.26)$$

$$y = 0; \quad u_{00} = 0, \quad u_{01} = 0, \quad u_{10} = 0, \quad u_{11} = 0,$$

$$\theta_{00} = 1, \quad \theta_{01} = 0, \quad \theta_{10} = 1, \quad \theta_{11} = 0,$$

$$y \to \infty; \quad u_{00} = 0, \quad u_{01} = 0, \quad u_{10} = 0, \quad u_{11} = 0,$$

$$\theta_{00} = 0, \quad \theta_{01} = 0, \quad \theta_{10} = 0, \quad \theta_{11} = 0.$$

(1.27)

Now, the above differential equations are linear and easily solvable. In view of equations (1.15) – (1.18), the equations (1.9) can be written as

$$u(y,\, t) = \left[u_{00}(y) + E\,u_{01}(y) + 0\left(E^2\right)\right]$$

$$+ \in e^{i\omega t}\left[u_{10}(y) + E\,u_{11}(y) + 0\left(E^2\right)\right],$$

(1.28)

$$\theta(y,\, t) = \left[\theta_{00}(y) + E\,\theta_{01}(y) + 0\left(E^2\right)\right]$$

$$+ \in e^{i\omega t}\left[\theta_{10}(y) + E\,\theta_{11}(y) + 0\left(E^2\right)\right].$$

(1.29)

As the second term of harmonic fluctuation in the above equations is the product of $\in E$ which can be neglected on order consideration, therefore we have

$$u(y,\ t) = u_{00}(y) + E\,u_{01}(y) + \in e^{i\omega t}\ u_{10}(y), \qquad (1.30)$$

$$\theta(y,\ t) = \theta_{00}(y) + E\ \theta_{01}(y) + \in e^{i\omega t}\ \theta_{10}(y). \qquad (1.31)$$

The values of u_{00}, u_{01}, u_{10}, θ_{00}, θ_{01} and θ_{10} can be obtained by solving the equations (1.19), (1.20), (1.21), (1.23) (1.24) and (1.25) respectively subject to the boundary conditions (1.27). Thus we get

$$u_{00} = \frac{G}{P^2 - P - \frac{M}{4}}\left(e^{-Ly} - P^{-Py}\right),$$

$$u_{01} = \frac{G^3}{2\left(P^2 - P\ \frac{M}{4}\right)^3}\left[\left\{\left(P + \frac{M}{4P}\right) + \frac{P}{2L - P}\left(L + \frac{M}{4L}\right)\right.\right.$$

$$\left. - \frac{4P}{L - P}\left(P + \frac{M}{4L}\right)\right\}\left(e^{-Ly} - e^{-Py}\right) + \frac{\left(P + \frac{M}{4P}\right)\left(P^2 - P - \frac{M}{4}\right)}{\left(4P^2 - 2P - \frac{M}{4}\right)}$$

$$\left(e^{-2Py} - e^{-Ly}\right) + \frac{P\left(L + \frac{M}{4L}\right)\left(P^2 - P - \frac{M}{4}\right)}{(2L - P)\left(4L^2 - 2L - \frac{M}{4}\right)}\left(e^{-2Ly} - e^{-Ly}\right)$$

$$+\frac{4P\left(P+\frac{M}{4L}\right)\left(P^{2}-P-\frac{M}{4}\right)}{(L-P)\left\{(P+L)^{2}-(P+L)-\frac{M}{4}\right\}}\left(e^{-Ly}-e^{-(P+Ly)}\right)\Bigg],$$

$$u_{10}=\frac{A\ G\ L}{\left(P^{2}-P-\frac{M}{4}\right)\left(L^{2}-L-\frac{M+i\omega}{4}\right)}\left(e^{-Ly}-e^{-Hy}\right)$$

$$+\frac{4\ i\ A\ PG}{\omega\left(P^{2}-P-\frac{M}{4}\right)}\left(e^{-Hy}-e^{-Py}\right)$$

$$+\frac{4\ i\ A\ P\ G}{\omega\left(n^{2}-n-\frac{M+i\omega}{4}\right)}\left(e^{-ny}-e^{-Hy}\right),$$

$$\theta_{00}=e^{-Py},$$

$$\theta_{01}=\frac{G^{2}}{\left(P^{2}-P-\frac{M}{4}\right)^{2}}\Bigg[\frac{1}{2}\left(P+\frac{M}{4P}\right)\left(e^{-Py}-e^{-2Py}\right)$$

$$+\frac{P\left(L+\frac{M}{4L}\right)}{2(2L-P)}\left(e^{-Py}-e^{-2Ly}\right)+\frac{2P\left(P+\frac{M}{4L}\right)}{(L-P)}\left(e^{-(P+L)y}\ e^{-Py}\right)\Bigg],$$

$$\theta_{10}=\frac{4\,i\,A\,P}{\omega}\left(e^{-Py}-e^{-ny}\right),$$

where $$L = \frac{1}{2}\left[1 + (1+M)^{\frac{1}{2}}\right],$$

$$H = \frac{1}{2}\left[1 + (1+M+i\,\omega)^{\frac{1}{2}}\right],$$

$$n = \frac{1}{2}\left[P + \left(P^2 + i\,\omega\,P\right)^{\frac{1}{2}}\right].$$

Putting the values of u_{00}, u_{01}, θ_{00} and θ_{01}, we obtain the mean velocity and mean temperature as

$$u_0(y) = u_{00} + E\; u_{01}(y)\,, \tag{1.32}$$

$$\theta_0(y) = \theta_{00} + E\; \theta_{01}(y)\,. \tag{1.33}$$

For $\omega t = \pi/2$, the transient velocity and transient temperature are

$$u\left(y, \frac{\pi}{2\omega}\right) = u_{00}(y) + E\; u_{01}(y) - \in u_i\,, \tag{1.34}$$

$$\theta\left(y, \frac{\pi}{2\omega}\right) = \theta_{00}(y) + E\; \theta_{01}(y) - \in \theta_i\,, \tag{1.35}$$

where, u_i and θ_i are the imaginary parts of u_{10} and θ_{10}, respectively.

From the equation (1.30) we calculate the skin friction which is given by

$$\tau = \left(\frac{du}{dy}\right)_{y=0}$$

$$= \tau_m + \in e^{i\omega t}\left[\frac{A\ G\ L(H-L)}{\left(P^2-P-\frac{M}{4}\right)\left(L^2-L-\frac{M+i\omega}{4}\right)}\right.$$

$$\left.+\frac{4iAPG(P-H)}{\omega\left(P^2-P-\frac{M}{4}\right)}+\frac{4iAPG(H-n)}{\omega\left(n^2-n-\frac{M+i\omega}{4}\right)}\right], \quad (1.36)$$

where, τ_m is the mean skin friction and is given by

$$\tau_m = \left(\frac{d u_0}{d y}\right)_{y=0}$$

$$= \frac{G(P-L)}{\left(P^2-P-\frac{M}{4}\right)} + \frac{E\ G^3}{2\left(P^2-P-\frac{M}{4}\right)^3}$$

$$\left[\left\{\left(P+\frac{M}{4P}\right) + \frac{P}{(2L-P)}\left(L+\frac{M}{4L}\right) - \frac{4P}{L-P}\left(P+\frac{M}{4L}\right)\right\}\right.$$

$$(P-L)+\frac{\left(P^2-P-\frac{M}{4}\right)\left(P+\frac{M}{4P}\right)(L-2P)}{4P^2-2P-\frac{M}{4}}$$

$$- \frac{P\left(L+\frac{M}{4L}\right)\left(P^{2}-P-\frac{M}{4}\right)L}{(2L-P)\left(4L^{2}-2L-\frac{M}{4}\right)}$$

$$\left. + \frac{4P^{2}\left(P+\frac{M}{4L}\right)\left(P^{2}-P-\frac{M}{4}\right)}{(L-P)\left\{(P+L)^{2}-(P+L)\frac{M}{4}\right\}}\right].$$

(1.37)

In terms of amplitude and phase, the skin friction can be expressed as

$$\tau = \tau_{m} + \in |B| \cos(\omega t + \alpha), \qquad (1.38)$$

where

$B = B_{r} + i\, B_{i} = \text{coefficient of } \in e^{i\omega t}$ in equation (1.34),

$$\tan\alpha = \frac{B_{i}}{B_{r}}.$$

From the equation (1.31), we calculate the rate of heat transfer from the plate to the fluid which is given by

$$K = -\left(\frac{d\theta}{dy}\right)_{y=0} = K_{m} + \in e^{i\omega t}\left(\frac{4i\, A\, P\, (P-n)}{\omega}\right)$$

(1.39)

where, K_m is the mean heat transfer rate and is given by

$$K_m = -\left(\frac{d\theta_0}{dy}\right)_{y=0}$$

$$= P - \frac{E\, G^2\, P}{\left(P^2 - P - \frac{M}{4}\right)^2}$$

$$\left[\frac{1}{2}\left(P + \frac{M}{4P}\right) + \frac{1}{2}\left(L + \frac{M}{4L}\right) - \frac{2L}{L-P}\left(P + \frac{M}{4L}\right)\right]. \tag{1.40}$$

In terms of amplitude and phase, K can be written as

$$K = K_m + \in |D| \cos(\omega t + \delta),$$

where

$D = D_r + i D_i =$ coefficient of $\in e^{i\omega t}$ in equation (1.39),

$$\tan \delta = \frac{D_i}{D_r}.$$

Results and Discussion:

For different values of G, E and P, the values of mean skin friction τ_m and mean heat transfer rate K_m are entered in Tables – 1 and 2.

By putting M = 0 in the equation (1.37), the values of τ_m for air are obtained to be negative for both G > 0 or G < 0 and moreover τ_m is dominated by the second term (coefficient of E) in the right hand side of the above equation. Due to the application of magnetic field such as M = 8, the values of τ_m are positive for G > 0 and negative for G < 0 and moreover τ_m is dominated by the first term of the equation (1.37). From the Table – 1 we observe that due to greater cooling of the plate, τ_m increases for air and water whereas owing to the greater viscous dissipative heat, τ_m decreases for air and increases for water. With increasing Prandtl number, τ_m decreases for G > 0 and increases for G < 0.

From Table –2 we observe that due to greater cooling of the plate, K_m increases for air and decreases for water whereas due to grater heating of the plate, K_m decreases for air and increases for water. For G > 0, greater viscous dissipative heat leads to an increase in K_m for air and a decrease in K_m for water while for G < 0, the conclusion is reverse. With increasing Prandtl number, K_m increases for both G less than or greater than 0.

Table –1 Mean skin friction, M = 8

G	E	P		
		0.71	**3.00**	**7.00**
5	0.01	2.8813	1.4388	0.6257
5	0.02	2.8387	1.6277	0.6265
10	0.01	5.5069	4.0110	1.2560
10	0.02	5.1659	5.5220	1.2620

15	0.01	7.6209	8.8496	1.8952
– 5	– 0.01	– 2.9665	– 1.0611	– 0.6242
– 5	– 0.02	– 3.0091	– 0.8722	– 0.6235
– 10	– 0.01	– 6.1889	– 0.9890	– 1.2440
– 10	– 0.02	– 6.5302	0.5252	– 1.2380
– 15	– 0.01	– 9.9228	1.3496	– 1.8547

Table –2 Mean rate of heat transfer, M = 8

G	**E**	**P**		
		0.71	**3.00**	**7.00**
5	0.01	0.7843	2.0937	6.9873
5	0.02	0.8587	1.1875	6.9747
10	0.01	1.0074	– 0.6250	6.9495
10	0.02	1.3048	– 4.2500	6.8990
15	0.01	1.3791	– 5.1562	6.8863
– 5	– 0.01	0.6356	3.9062	7.0126
– 5	– 0.02	0.5613	4.8125	7.0252

– 10	– 0.01	0.4126	6.6250	7.0505
– 10	– 0.02	0.1152	10.2500	7.1010
– 15	– 0.01	0.0408	11.1562	7.1136

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