New Sort of functions in Nano Topology

B. Meera Devi [1] & T. Nithya [2]

[1] *Assistant Professor,*
PG and Research Department of Mathematics,
Sri S.R.N.M.College, Sattur-626 203,
Tamil Nadu, India.

[2] *Research Scholar*
PG and Research Department of Mathematics,
Sri S.R.N.M.College, Sattur-626 203, Tamil Nadu, India.

Abstract

In this paper we introduce a new class of functions called $N\delta\hat{g}$ -continuous functions. We obtain several characterizations and some their properties. Also we investigate its relationship with other existing functions. Further we introduce and study a new class of functions namely $N\delta\hat{g}$ -closed map.

Keywords: *Nδg -closed sets, Nδg -continuous function, $N\delta\hat{g}$-closed sets, $N\delta\hat{g}$-continuous function, $N\delta\hat{g}$-closed maps.*

AMS Subject Classification: *54A05, 54C08*

1. Introduction

M. Lellis Thivagar and Carmel Richard [7] introduced nano topological space (or simply NTS) with respect to a subset X of a universe which is defined in terms of lower and upper approximation of X. He has also defined nano closed sets (briefly N-CS), nano interior and nano closure of a set. M. Lellis Thivagar and Carmel Richard [7] introduced nano semi-open, nano regular-open, nano pre open, nano α -open. M. Y Bakeir [1], V. Pankajam and K. Kavitha [9], K. Bhuvaneswari and K. Mythili Gnanapriya [4] introduced Nδg-closed, Nδ-continuous, Nδ-closed map, Ng-closed, Ng-continuous respectively. The purpose of this present paper is to define a new class of nano generalized continuous function called $N\delta\hat{g}$-continuous functions and investigate their relationships to other generalized continuous functions. We further introduce and study a new class of functions namely $N\delta\hat{g}$-closed map.
N.

2 Preliminaries

Throughout this paper, (U, τ_R(X)), (V, σ_R(Y)) and (W, η_R(Z)) represent Nano Topological Spaces on which no separation axioms are assumed unless or otherwise mentioned. For a set A in a NTS (U, τ_R(X)), Ncl(A), Nint(A) and A^c denote the nano closure of A, the nano interior of A and the nano complement of A respectively. Let us recall the following definitions, which are useful in the sequel.

Definition 2.1. [7] Let U be a non-empty finite set of objects called the universe and R be an equivalence relation on U named as the indiscernibility relation. Elements belonging to the same equivalence class are said to be indiscernible with one another. The pair (U, R) is said to be the approximation space. Let X⊆U.

(i) The lower approximation of X with respect to R is the set of all objects, which can be for certain classified as X with respect to R and it is denoted by L_R(X). That is $L_R(X) = \cup_{x\in U}\{R(x) : R(x) \subseteq X , x \in U\}$, where R(x) denotes the equivalence class determined by x .

(ii) The upper approximation of X with respect to R is the set of all objects, which can be possibly classified as X with respect to R and its is denoted by U_R(X) . That is, $U_R(X) = \cup_{x\in U}\{R(x) : R(x) \cap X \neq \phi , x \in U\}$.

(iii) The boundary region of X with respect to R is the set of all objects, which can be classified neither as X nor as not X with respect to R and it is denoted by B_R(X) . That is, $B_R(X) = U_R(X) - L_R(X)$.

Proposition 2.2. [10] If (U, R) is an approximation space and X , Y ⊆ U , then

(i) $L_R(X) \subseteq X \subseteq U_R(X)$

(ii) $L_R(\phi) = U_R(\phi)$ and $L_R(U) = U_R(U) = U$

(iii) $U_R(X \cup Y) = U_R(X) \cup U_R(Y)$

(iv) $U_R(X \cap Y) \subseteq U_R(X) \cap U_R(Y)$

(v) $L_R(X) \cup L_R(Y) \subseteq L_R(X \cup Y)$

(vi) $L_R(X \cap Y) = L_R(X) \cap L_R(Y)$

(vii) $L_R(X) \subseteq L_R(Y)$ and $U_R(X) \subseteq U_R(Y)$ whenever $X \subseteq Y$

(viii) $U_R(X^c) = [L_R(X)]^c$ and $L_R(X^c) = [U_R(X)]^c$

(ix) $U_R[U_R(X)] = L_R[U_R(X)] = U_R(X)$

(x) $L_R[L_R(X)] = U_R[L_R(X)] = L_R(X)$

Definition 2.3. [7] Let U be the universe, R be an equivalence relation on U and $\tau_R(X) = \{\phi, U, L_R(X), U_R(X), B_R(X)\}$ where X⊆ U . Then by proposition 2.2 τ_R(X) satisfies the following axioms.

(i) U and $\phi \in \tau_R$(X) .

(ii) The union of the elements of any subcollection of τ_R(X) is in τ_R(X) .

(iii) The intersection of the elements of any finite subcollection of τ_R(X) is in τ_R(X) .

That is, τ_R(X) forms a topology on U called as the nano topology on U with respect to X . We call (U, τ_R(X)) as the nano topological space. The elements of τ_R(X) are called as nano open sets (briefly N-OS).

Definition 2.4. [7] If (U, τ_R(X)) is a Nano topological space with respect to X where X⊆U and if A⊆U, then
(i) The Nano interior of A is defined as the union of all Nano-open subsets of A and it is denoted by NInt(A). That is, NInt(A) is the largest Nano-open subset of A .
(ii) The Nano closure of A is defined as the intersection of all Nano closed sets containing A and it is denoted by Ncl(A). That is, Ncl(A) is the smallest Nano closed set containing A .

Definition 2.5. [7] Let (U, τ_R(X)) be a Nano topological space and A⊆U. Then A is said to be
(i) Nano semi open if A⊆NCl[NInt(A)] .
(ii) Nano semi closed if NInt[NCl(A)]⊆ A .
(iii) Nano pre open if A⊆NInt[N Cl(A)] .
(iv) Nano α open if A⊆ NInt[NCl(N Int(A))] .

NSO (U, X), NSC (U, X), NPO (U, X) and NαO (U, X) respectively denote the families of all Nano semi open, Nano semi closed, Nano pre-open and Nano α open subsets of U .

Definition 2.6. [1] The Nδ−interior of a subset A of U is the union of all nano regular open set of U contained in A and is denoted by $NInt_\delta$(A). The subset A is called Nδ−open if A = $NInt_\delta$(A), i.e. a set is Nδ−open if it is the union of nano regular open sets. The complement of a Nδ−open is called Nδ−closed. Alternatively, a set A ⊆(U, τ_R(X)) is called Nδ−closed if A=Ncl_δ(A), where Ncl_δ(A)={x∈U: NInt(Ncl(M))∩A $\neq \phi$, M∈ τ_R(X) and x∈ M }.

Definition 2.7. A subset A of a NTS (U, τ_R(X)) is called
(i) nano g-closed set (briefly Ng − CS) [4] if Ncl(A)⊆M whenever A ⊆ M and M is a N – OS in (U, τ_R(X)) .
(ii) nano sg − closed set (briefly Nsg -CS) [2] if Nscl(A) ⊆M whenever A ⊆ M and M is a N s -OS in (U, τ_R(X)).
(iii) nano gs − closed set (briefly Ngs -CS) [2] if Nscl(A) ⊆M whenever A ⊆M and M is a N –OS in (U, τ_R(X)).
(iv) nano αg − closed set (briefly Nαg -CS) [12] if Nαcl(A) ⊆ M whenever A ⊆ M and M is a N − OS in (U, τ_R(X)).
(v) nano gα − closed set (briefly Ngα -CS) [12] if Nαcl(A) ⊆ M whenever A ⊆M and M is a Nα – OS in (U, τ_R(X)).
(vi) nano δg− closed set (briefly Nδg -CS) [1] if Ncl_δ(A)⊆M whenever A⊆M and M is nano open set in (U, τ_R(X)).
(vii) nano δ$\hat{g}$− closed set (briefly Nδ$\hat{g}$ -CS) [8] if Ncl_δ(A)⊆M whenever A⊆M and M is a nano $\hat{g}$ open set in (U , τ_R(X)).

The complement of a (resp. Ng-closed, Nsg-closed, Ngs-closed, Nαg-closed, Ngα-closed, Nδg-closed and N$\delta\hat{g}$-closed) set is called (resp. Ng-open, Nsg – open, Ngs -open, Nαg -open, Ngα -open, Nδg -open and N$\delta\hat{g}$ -open).

Definition 2.8. [6] Let (U, τ_R(X)) and (V, $\tau_{R'}$(Y)) be two Nano topological spaces. Then a mapping f :(U, τ_R(X))→(V , $\tau_{R'}$(Y)) is Nano continuous on U if the inverse image of every Nano open set in V is Nano open in U .

Definition 2.9. A function f : (U , τ_R(X)) → (V , $\tau_{R'}$(Y)) is called

(i) Ng -continuous [5] if the inverse image of every Nano open set in V is nano g-open in U.

(ii) Nαg -continuous [12] if f^{-1}(W) is Nαg -closed in (U, τ_R(X)) for every nano closed set W in (V , $\tau_{R'}$(Y)) .

(iii) Ngα -continuous [12] if f^{-1}(W) is Ngα -closed in (U, τ_R(X)) for every nano closed set W in (V , $\tau_{R'}$(Y)) .

(iv) Nδ -continuous [9] if f^{-1}(W) is Nδ -closed in (U, τ_R(X)) for every nano closed set W in (V , $\tau_{R'}$(Y)) .

(v) Nsg-continuous [3] if f^{-1}(W) is Nsg -closed in (U, τ_R(X)) for every nano closed set W in (V , $\tau_{R'}$(Y)) .

Definition 2.10. A function f : (U , τ_R(X)) → (V , $\tau_{R'}$(Y)) is called

(i) Nano closed map [6] if f(W) is Nano closed in (V, $\tau_{R'}$ (Y)) for every nano closed set W in (U, τ_R(X)).

(ii) Ng closed map [5] if f(W) is Ng closed in (V, $\tau_{R'}$ (Y)) for every nano closed set W in (U, τ_R(X)).

(iii) Nδ closed map [9] if f(W) is Nδ closed in (V, $\tau_{R'}$ (Y)) for every nano closed set W in (U, τ_R(X)).

(iv) Nαg closed map [12] if f(W) is Nαg closed in (V, $\tau_{R'}$ (Y)) for every nano closed set W in (U, τ_R(X)).

(v) Ngs closed map [2] if f(W) is Ngs closed in (V, $\tau_{R'}$ (Y)) for every nano closed set W in (U, τ_R(X)).

3. N$\delta\hat{g}$ Continuous Maps and N$\delta\hat{g}$ Closed Maps

We introduce the following definition.

Definition 3.1. A function f : (U, τ_R(X))→(V, σ_R(Y)) is said to be N$\delta\hat{g}$ –continuous map if f^{-1}(W) is N$\delta\hat{g}$ -closed in (U , τ_R(X)) for every nano closed set W of (V, σ_R(Y)).

Example 3.2. Let U={a, b, c} with U/R = {{a}, {b, c}, {b}} and X = {a , b} with nano topology τ_R(X) = {U, ϕ, {a, b}, {c}}. Let V={x, y, z} with V/R = {{x}, {y, z}, {y}} and Y = {x, y} with nano topology σ_R(Y)={V, ϕ, {x, y}, {z}}. Define a function f : (U, τ_R(X))→(V, σ_R(Y)) by f(a) = x , f(b) = y , f(c) = z . Clearly f is N$\delta\hat{g}$ –continuous.

Theorem 3.3. Every N$\delta\hat{g}$ –continuous function is Nδg-continuous.

Proof. It is true that every N$\delta\hat{g}$ –closed set is Nδg-closed.

Remark 3.4. The converse of the above the theorem is not true in general as shown in the following example.

Example 3.5. Let U={a, b, c} with U/R = {{a}, {b, c}, {b}} and X = {a , b} with nano topology τ_R(X)={U, ϕ, {a, b}, {c}}. N$\delta\hat{g}$–closed={U, ϕ, {a, b}, {c}}. Let V={p, q, r} with V/R = {{p}, {q, r}, {q}} and Y = {p, r} with nano topology σ_R(Y) = {V, ϕ, {q, r}, {p}}. Define a function f : (U, τ_R(X))→ (V, σ_R(Y)) by f(a)=p, f(b) = q , f(c) = r. Then f is not N$\delta\hat{g}$ –continuous because f^{-1}\{p} = {a} is not N$\delta\hat{g}$ –closed in (U, τ_R(X)). But f is Nδg –continuous.

Theorem 3.6. Every N$\delta\hat{g}$-continuous function is Ng -continuous.
Proof. It is true that every N$\delta\hat{g}$-closed set is Ng -closed.

Remark 3.7. The converse of the above theorem is not true in general as shown in the following example.

Example 3.8. Let U= {a, b, c} with U/R = {{a}, {b, c}, {b}} and X = {a, b} with nano topology τ_R(X)={U, ϕ, {a, b}, {c}}. Let V= {a, b, c} with V/R = {{a}, {b, c}} and Y = {a, c} with nano topology σ_R(Y)={V, ϕ, {b, c}, {a}}. Define a function f : (U, τ_R(X))→ (V, σ_R(Y)) by f(a) = b, f(b)=a, f(c) = c. Then f is not N$\delta\hat{g}$ –continuous function because {a} is nano closed in (V, σ_R(Y)) but f^{-1}{a}={b} is not N$\delta\hat{g}$ –closed in (U, τ_R(X)). But f is Ng –continuous.

Theorem 3.9. Every Nδ-continuous function is N$\delta\hat{g}$ -continuous.

Proof. It is true that every Nδ-closed set is N$\delta\hat{g}$ -closed. □

Remark 3.10. The converse of the above theorem is not true in general as shown in the following example.

Example 3.11. Let U = {a, b, c} with U/R = {{a , c} , {b}} and X = {b} with nano topology τ_R(X)={U , ϕ, {b}}. Let V= {a, b, c} with V /R = {{a}, {a, b}, {c}} and Y = {a , c} with nano topology σ_R(Y) = {V, ϕ, {a, c}, {b}}. Define a function f : (U , τ_R(X)) → (V, σ_R(Y)) by f(a)=a , f(b)=c , f(c)=b. Then f is not Nδ -continuous function because {b} is Nδ-open in (V, σ_R(Y)) but f^{-1}{b} = {c} is not Nδ -open in (U , τ_R(X)) . However f is N$\delta\hat{g}$ -continuous function.

Theorem 3.12. Every N$\delta\hat{g}$ -continuous function is Ngs -continuous.

Proof. It is true that every N$\delta\hat{g}$-closed set is Ngs -closed.

Remark 3.13. The converse of the above theorem is not true in general as shown in the following example.

Example 3.14. Let U = { a, b, c} with U/R = {{b , c}, {a}} and X = {a, c} with nano topology τ_R(X) = {U , ϕ, {a}, {b, c}}. Let V = {x, y, z} with V /R ={x, y}, and Y = {x} with nano topology σ_R(Y) = {V, ϕ , {x, y}}. Define a function f : (U , τ_R(X)) → (V, σ_R(Y)) by f(a)=x, f(b)=z, f(c)=y. Then f is Ngs-continuous. But f is not Nδ$\hat{g}$-continuous function because {z} is nano closed set in (V, σ_R(Y)) but f^{-1}{z}={b} is not Nδ$\hat{g}$-closed in (U, τ_R(X)). However f is Nδ$\hat{g}$-continuous function.

Theorem 3.15. Every Nδ$\hat{g}$-continuous function is Nαg -continuous.

Proof. It is true that every Nδ$\hat{g}$-closed set is Nαg -closed.

Remark 3.16. The converse of the above theorem is not true in general as shown in the following example.

Example 3.17. Let U = {a, b, c} with U/R = {{b , c} , {a} , {b}} and X = {a , b} with nano topology τ_R(X) = {U , ϕ , {c} , {a , b}} . Let V = {x , y, z} with V /R = {{x}, {y, z}} and Y = {x} with nano topology σ_R(Y) = {V, ϕ , {x}}. Define a function f : (U, τ_R(X)) → (V, σ_R(Y)) by f(a) = x, f(b) = y, f(c) = z. Then f is Nαg -continuous. But f is not Nδ$\hat{g}$ continuous function. Since for the nano closed set {y , z} of (V, σ_R(Y)) but f^{-1}{y , z} = {b , c} is not Nδ$\hat{g}$ -closed in (U , τ_R(X)).

Remark 3.18. The following example shows that Nδ$\hat{g}$-continuous is independent from N$\hat{g}$-continuous, Nsg-continuous and Ngα −continuous.

Example 3.19. Let U = { a, b, c } with U/R = {{b , c} , {a}} and X = {a} with nano topology τ_R(X) = {U , ϕ, {a} }. Let V = {x , y, z} with V /R = {{y}, {x , z}} and Y = {y} with nano topology σ_R(Y) = {V, ϕ, {y}}. Define a function f : (U, τ_R(X)) → (V, σ_R(Y)) by f(a) = x, f(b) = y, f(c) = z. Then f is Nδ$\hat{g}$-continuous. But f is not N$\hat{g}$ continuous function, Nsg-continuous and Ngα- continuous function. Since for the nano closed set {x, z} of (V, σ_R(Y)), f^{-1}{x , z} = {a, c} is not N$\hat{g}$-closed, Nsg-closed and Ngα-closed in (U , τ_R(X)).
Also the another example, Let U={a, b, c} with U/R = {{a}, {a, b}, {c}} and X = {a , c} with nano topology τ_R(X)={U, ϕ, {a, c}, {b}}. Let V= {p, q, r} with V/R = {{p}, {q, r}, {q}} and Y = {p, q} with nano topology σ_R(Y)={V, ϕ, {p, q}, {r}}. Define a function f : (U, τ_R(X))→(V, σ_R(Y)) by f(a) = p , f(b) = q , f(c) = r. Then f is N$\hat{g}$ continuous, Nsg-continuous and Ngα- continuous. But f is not Nδ$\hat{g}$-continuous function. Since for the nano closed set {r} of (V, σ_R(Y)), f^{-1}{r} = {c} is not Nδ$\hat{g}$ –closed in (U, τ_R(X)).

Remark 3.20. The composition of two Nδ$\hat{g}$ -continuous function need not be Nδ$\hat{g}$-continuous as the following example shows.

Example 3.21. Let U = {a , b, c} with U/R = {a , b} and X = {a} with nano topology $\tau_R(X)$ = {U , ϕ , {a , b}}. Let V = {a , b, c} with V /R={{a , c}, {b}} and Y = {b} with nano topology $\sigma_R(Y)$ = {V, ϕ, {b}}. Let W={p, q, r} with W/R ={{q, r}, {p}} and Z = {p} with nano topology $\eta_R(Z)$={W, ϕ, {p}}. Define a function f: (U, $\tau_R(X)$)→(V , $\sigma_R(Y)$) by f(a)= b, f(b)=c, f(c)=a and let g: (V, $\sigma_R(Y)$) → (W, $\eta_R(Z)$) by g(a)=p, g(b)=q, g(c)=r. Clearly f and g are N$\delta\hat{g}$ –continuous function. But g∘f : (U, $\tau_R(X)$)→(W, $\eta_R(Z)$) is not an N$\delta\hat{g}$ –continuous function because $(g \circ f)^{-1}$ ({q , r}) = $f^{-1}[g^{-1}(\{q , r\})]$ = $f^{-1}(\{b , c\})$ = {a , b} is not an N$\delta\hat{g}$ -closed set of (U, $\tau_R(X)$) where {q , r} is a nano closed set of (W , $\eta_R(Z)$).

Definition 3.22. A map f : (U, $\tau_R(X)$)→(V, $\sigma_R(Y)$) is called N$\delta\hat{g}$ - closed map if the image of each nano closed set in (U, $\tau_R(X)$) is N$\delta\hat{g}$ -closed in (V, $\sigma_R(Y)$) .

3.23. Every N$\delta\hat{g}$-closed map is Ng -closed map.

Proof. It is true that every N$\delta\hat{g}$-closed set is Ng -closed.

Remark 3.24. The converse of the above theorem is not true in general as shown in the following example.

Example 3.25. Let U = {a, b, c} with U/R = {{a}, {b , c}, {b}} and X={a, b} with nano topology $\tau_R(X)$ = {U , ϕ, {a, b} , {c}}. Let V = {x , y, z} with V /R ={{x}, {y, z}} and Y = {x , z} with nano topology $\sigma_R(Y)$={V, ϕ, {y, z}, {x}}. Ng-closed {V, ϕ, {x}, {y}, {z}, {x, y}, {x, z} ,{y, z}}. N$\delta\hat{g}$ -closed {V, ϕ, {y, z}, {x}}. Define a function f : (U, $\tau_R(X)$) → (V, $\sigma_R(Y)$) by f(a)=x, f(b) = y, f(c) = z. Here f is Ng -closed map, but not a N$\delta\hat{g}$ -closed map. Since the image of a nano closed set {a, b} in (U, $\tau_R(X)$), f^{-1} {a , b} = {x , y} is not N$\delta\hat{g}$– closed in (V, $\sigma_R(Y)$).

Theorem 3.26. Every N$\delta\hat{g}$ -closed map is Ngs -closed map.

Proof. It is true that every N$\delta\hat{g}$ -closed set is Ngs -closed. □

Remark 3.27. The converse of the above theorem is not true in general from the following example shows.

Example 3.28. Let U = { a, b, c } with U/R = {{a} , {b, c}} and X = {a} with nano $\tau_R(X)$ = {U, ϕ, {a}}. Let V = {a , b, c} with V /R = {{c}, {a, b}} and Y = {a, c} with nano topology $\sigma_R(Y)$={V, ϕ, {a, b}, {c}}. Define a function f : (U, $\tau_R(X)$) → (V, $\sigma_R(Y)$) by f(a)=b, f(b) = a, f(c) = c . Then clearly f is Ngs-closed map. It is not a N$\delta\hat{g}$ –closed map because {b, c} is nano closed in (U, $\tau_R(X)$) but f{b, c}={a, c} is not N$\delta\hat{g}$ –closed in (V, $\sigma_R(Y)$) .

Theorem 3.29. Every N$\delta\hat{g}$ -closed map is Nδg -closed map.

Proof. It is true that every N$\delta\hat{g}$ -closed set is Nδg -closed. □

Remark 3.30. The converse of the above theorem is not true in general from the following example shows.

Example 3.31. Let U = {a, b, c} with U/R = {{a}, {b , c}, {b}} and X={a , b} with nano topology τ_R(X) = {U, ϕ, {a , b} , {c}}. Let V = {x, y, z} with V /R = {{x}, {y , z}} and Y = {x , z} with nano topology σ_R(Y) = {V, ϕ, {y, z}, {x}}. Define a function f: (U, τ_R(X)) → (V , σ_R(Y)) by f(a) = y, f(b) = x, f(c) = z. Here f is Nδg -closed map but is not a N$\delta\hat{g}$ -closed map since {c} is nano closed in (U, τ_R(X)), f{c} = {z} is not N$\delta\hat{g}$ -closed in (V, σ_R(Y)) .

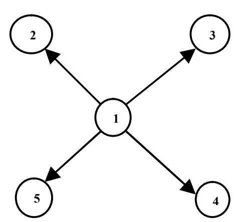
Theorem 3.32. Every N$\delta\hat{g}$-closed map is Nαg -closed map.

Proof. It is true that every N$\delta\hat{g}$ -closed set is Nαg -closed. □

Remark 3.33. The converse of the above theorem is not true in general from the following example shows.

Example 3.34. Let U = {a, b, c} with U/R = {{a, c}, {b}} and X={b} with nano topology τ_R(X)={U, ϕ, {b}}. Let V = {p, q, r} with V /R= {{p}, {q, r}, {q}} and Y={p, q} with nano topology σ_R(Y)={V, ϕ, {p, q}, {r}}. Define a function f: (U, τ_R(X))→(V, σ_R(Y)) by f(a) = p, f(b) = q, f(c) = r. Here f is Nαg-closed map but is not a N$\delta\hat{g}$ -closed map since {a, c} is nano closed in (U, τ_R(X)), f{a, c} = {p, r} is not N$\delta\hat{g}$ -closed in (V, σ_R(Y)) .

Remark 3.35. The following diagram shows the relationships of N$\delta\hat{g}$ -closed map with other known existing nano closed map. A → B represents A implies but is not conversely.



1. N$\delta\hat{g}$-closed map, 2. Nδg -closed map, 3. Nαg-closed map,
4. Ngs-closed map, 5. Ng-closed map.

Theorem 3.36. A map f : (U, τ_R(X))→(V, σ_R(Y)) is N$\delta\hat{g}$– closed if and only if for each subset G of (V, σ_R(Y)) and for each nano open set A of (U, τ_R(X)) Containing f^{-1}(G), there exists an N$\delta\hat{g}$–open set B of (V, σ_R(Y)) such that G⊂B and f^{-1}(B)□A.

Proof. Let f be an $N\delta\hat{g}$– closed map and let G be an subset of $(V, \sigma_R(Y))$ and A be an nano open set of $(U, \tau_R(X))$ containing $f^{-1}(G)$. Then U - A is nano closed in $(U, \tau_R(X))$. Since f is $N\delta\hat{g}$– closed map, f(U - A) is $N\delta\hat{g}$ -closed set in $(V, \sigma_R(Y))$. Hence V- f(U - A) is $N\delta\hat{g}$–open set in $(V, \sigma_R(Y))$. Take B = V- f (U - A). Then B is $N\delta\hat{g}$-open set in $(V, \sigma_R(Y))$ containing G such that $f^{-1}(B)$□A.
Conversely, let F be an nano closed subset of $(U, \tau_R(X))$. Then $f^{-1}(V - f(F))$ □ $U - F$ and $U - F$ is nano open. By hypothesis there is an $N\delta\hat{g}$-open set B of $(V, \sigma_R(Y))$ such that $V- f(F)$□B and $f^{-1}(B)$□$U - F$. Therefore, F□$U - f^{-1}(B)$. Hence $V - B$□ f(F)□$f(U - f^{-1}(B)) \subset V - B$ which implies $f(F) = V - B$ and hence f(F) is $N\delta\hat{g}$ -closed in $(V, \sigma_R(Y))$. Thus f is an $N\delta\hat{g}$-closed map. □

Theorem 3.37. Let $f : (U, \tau_R(X)) \to (V, \sigma_R(Y))$ and $g : (V, \sigma_R(Y)) \to (W, \eta_R(Z))$ be any two maps and $g \circ f : (U, \tau_R(X)) \to (W, \eta_R(Z))$ be an $N\delta\hat{g}$-closed map. If f is nano continuous then g is $N\delta\hat{g}$-closed.

Proof. Let B be nano closed in $(V, \sigma_R(Y))$. Since f is nano continuous, $f^{-1}(B)$ is nano closed in $(U, \tau_R(X))$. Since $g \circ f$ is $N\delta\hat{g}$-closed, $(g \circ f)(f^{-1}(B))$ is $N\delta\hat{g}$-closed in $(W, \eta_R(Z))$. That is g(B) is $N\delta\hat{g}$-closed in $(W, \eta_R(Z))$. Hence g is $N\delta\hat{g}$-closed.

Theorem 3.38. A bijection $f : (U, \tau_R(X)) \to (V, \sigma_R(Y))$ is $N\delta\hat{g}$-closed map iff f(A) is $N\delta\hat{g}$-open in $(V, \sigma_R(Y))$ for every nano open set A in $(U, \tau_R(X))$.

Proof. Let $f : (U, \tau_R(X)) \to (V, \sigma_R(Y))$ is $N\delta\hat{g}$-closed map and A be an nano open set in $(U, \tau_R(X))$. Then A^c is nano closed in $(U, \tau_R(X))$. Since f is $N\delta\hat{g}$-closed map, $f(A^c)$ is $N\delta\hat{g}$-closed set in $(V, \sigma_R(Y))$. But $f(A^c)=[f(A)]^c$ and hence $[f(A)]^c$ is $N\delta\hat{g}$-closed set in $(V, \sigma_R(Y))$. Hence f(A) is $N\delta\hat{g}$-open in $(V, \sigma_R(Y))$.
Conversely, f(A) is $N\delta\hat{g}$-open in $(V, \sigma_R(Y))$ for every nano open set A of $(U, \tau_R(X))$ then A^c is nano closed set in $(U, \tau_R(X))$ and $[f(A)]^c$ is a $N\delta\hat{g}$-closed in $(V, \sigma_R(Y))$. But $[f(A)]^c=f(A^c)$ and hence $f(A^c)$ is $N\delta\hat{g}$-closed in $(V, \sigma_R(Y))$. Therefore, f is $N\delta\hat{g}$-closed map.

References

[1] M. Y. Bakeir, "Nano delta generalized closed sets via nano topology", 17th International Conference on Differential Geometry and Application: ICDGA 2015, 2016.

[2] K. Bhuvaneswari and A. Ezhilarasi, "On nano semi-generalized and nano generalized semi closed sets in nano topological spaces", International Journal of Mathematics and Computer Applications Research, 4(3), 2014, pp: 117-124.

[3] K. Bhuvaneswari and A. Ezhilarasi, "On nano semi generalized continuous maps in nano topological spaces", International research journal of pure algebra 5(9), 2015, pp:149-155.

[4] K. Bhuvaneswari and k. Mythili Gnanapriya, "Nano generalized closed sets in nano topological spaces", International Journal of Scientific and Research Publications, Volume 4, Issue 5, 2015.

[5] K. Bhuvaneswari and k. Mythili Gnanapriya, "Nano generalized continuous function in nano topological space", International Journal of Mathematical Archive-6(6), 2015.

[6] M. Lellis Thivagar and Carmel Richard, "On nano continuity", Mathematical Theory and Modeling, Vol 3, No. 7, 2013.

[7] M. Lellis Thivagar and Carmel Richard, "On nano forms of weakly open sets", International Journal of Mathematics and Statistics Invention, 1(1)., 2013, pp. 31-37

[8] B. Meera Devi and T. Nithya, "$N\delta\hat{g}$-closed sets in nano topological spaces", (Communicated).

[9] K. Pankajam and K. Kavitha, " δ -open sets and δ -continuity in δ -nano topological spaces", International Journal of Innovative Science and Research Technology, Vol 2, 2017.

[10] Reilly. I. L and Vamanamurthy, "On α -sets in topological Spaces", Tamkang. J. Math, 16, 1985, pp: 7-11

[11] R. Thanga Nachiyar and K. Bhuvaneswari, "Nano generalized A- continuous and Nano A-generalized continuous functions in nano topological spaces", International Journal of Mathematics Trends and Technology, Vol. 14, 2014.

[12] *R. Thanga Nachiyar and K. Bhuvaneswari, "On nano generalized α - closed sets and nano α - generalized closed sets in nano topological spaces", International Journal of Engineering Trends and Technology, 13(6), 2014, pp: 257-260.*