$\widehat{\beta}\, g$ Closed Mappings in Intuitionistic Fuzzy Topological Spaces

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Abstract

In this paper we have introduced intuitionistic fuzzy $\widehat{\beta}$ generalized closed mappings and intuitionistic fuzzy $\widehat{\beta}$ generalized open mappings. Some of their basic properties are studied and we also introduce intuitionistic fuzzy i$\widehat{\beta}$ - generalized Closed mappings. We provide the relation between intuitionistic fuzzy i$\widehat{\beta}$ - generalized Closed mappings and intuitionistic fuzzy $\widehat{\beta}$ generalized Closed mappings.

Key words: *Intuitionistic fuzzy topology, intuitionistic fuzzy $\widehat{\beta}$ generalized closed sets, intuitionistic fuzzy $\hat{\beta}$ generalized closed mappings.*

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1. Introduction

The concept of fuzzy sets was introduced by Zadeh [11] and later Atanassov [1] generalized this idea to intuitionistic fuzzy sets using the notion of fuzzy sets. On the other hand Coker [3] introduced intuitionistic fuzzy topological spaces using the notion of intuitionistic fuzzy sets. In this paper, we have introduce intuitionistic fuzzy $\widehat{\beta}$ generalized closed mappings and studied some of their basic properties. We arrive at some characterizations of intuitionistic fuzzy $\widehat{\beta}$ generalized closed mappings and intuitionistic fuzzy $\widehat{\beta}$ generalized open mappings. We also introduce intuitionistic fuzzy i$\widehat{\beta}$ generalized Closed mappings.. We provide the relation between intuitionistic fuzzy i$\widehat{\beta}$ - generalized Closed mappings and intuitionistic fuzzy $\widehat{\beta}$ generalized Closed mappings.

2. Preliminaries

Definition 2.1: [1] Let X be a non-empty fixed set. An intuitionistic fuzzy set (IFS in short) A in X is an object having the form

$$A = \{\langle x, \mu_A(x), \nu_A(x) \rangle / x \in X\}$$

where the functions $\mu_A(x): X \to [0, 1]$ and $\nu_A(x): X \to [0, 1]$ denote the degree of membership (namely $\mu_A(x)$) and the degree of non -membership (namely $\nu_A(x)$) of each element $x \in X$ to the set A, respectively, and $0 \leq \mu_A(x) + \nu_A(x) \leq 1$ for each $x \in X$. Denote the set of all intuitionistic fuzzy sets in X by IFS (X).

Definition 2.2: [1] Let A and B be IFSs of the form

$A = \{\langle x, \mu_A(x), \nu_A(x) \rangle / x \in X \}$ and $B = \{ \langle x, \mu_B(x), \nu_B(x) \rangle / x \in X \}$. Then

(a) $A \subseteq B$ if and only if $\mu_A(x) \leq \mu_B(x)$ and $\nu_A(x) \geq \nu_B(x)$ for all $x \in X$

(b) $A = B$ if and only if $A \subseteq B$ and $B \subseteq A$

(c) $A^c = \{ \langle x, \nu_A(x), \mu_A(x) \rangle / x \in X \}$

(d) $A \cap B = \{ \langle x, \mu_A(x) \wedge \mu_B(x), \nu_A(x) \vee \nu_B(x) \rangle / x \in X \}$

(e) $A \cup B = \{ \langle x, \mu_A(x) \vee \mu_B(x), \nu_A(x) \wedge \nu_B(x) \rangle / x \in X \}$

For the sake of simplicity, we shall use the notation $A = \langle x, \mu_A, \nu_A\rangle$ instead of $A = \{ \langle x, \mu_A(x), \nu_A(x) \rangle / x \in X \}$. Also forthe sake of simplicity, we shall use the notation $A = \{ \langle x, (\mu_A, \mu_B), (\nu_A, \nu_B) \rangle \}$ instead of $A = \langle x, (A/\mu_A, B/\mu_B), (A/\nu_A, B/\nu_B) \rangle$.

The intuitionistic fuzzy sets $0_\sim = \{ \langle x, 0, 1 \rangle / x \in X \}$ and $1_\sim = \{\langle x, 1, 0 \rangle / x \in X\}$ are respectively the empty set and the whole set of X.

Definition 2.3: [3] An intuitionistic fuzzy topology (IFT in short) on X is a family τ of IFSs in X satisfying the following axioms.

(i) $0_\sim, 1_\sim \in \tau$
(ii) $G_1 \cap G_2 \in \tau$ for any $G_1, G_2 \in \tau$

(iii) $\cup G_i \in \tau$ for any family $\{ G_i / i \in J \} \subseteq \tau$.

In this case the pair (X, τ) is called an intuitionistic fuzzy topological space (IFTS in short) and any IFS in τ is known as an intuitionistic fuzzy open set (IFOS in short) in X.

The complement A^c of an IFOS A in IFTS (X, τ) is called an intuitionistic fuzzy closed set (IFCS in short) in X.

Definition 2.4:[3] Let (X, τ) be an IFTS and $A = \langle x, \mu_A, \nu_A \rangle$ be an IFS in X. Then the intuitionistic fuzzy interior and intuitionistic fuzzy closure are defined by

$int(A) = \cup \{ G / G$ is an IFOS in X and $G \subseteq A \}$,

$cl(A) = \cap \{ K / K$ is an IFCS in X and $A \subseteq K \}$.

Note that for any IFS A in (X, τ), we have $cl(A^c) = [int(A)]^c$ and $int(A^c) = [cl(A)]^c$.

Definition 2.5:[7] An IFS $A = \{ \langle x, \mu_A, \nu_A \rangle \}$ in an IFTS (X, τ) is said to be an

(i) intuitionistic fuzzy semi open set (IFSOS in short) if $A \subseteq cl(int(A))$,

(ii) intuitionistic fuzzy α-open set (IFαOS in short) if $A \subseteq int(cl(int(A)))$,

(iii) intuitionistic fuzzy regular open set (IFROS in short) if $A = int(cl(A))$.

The family of all IFOS (respectively IFSOS, IFαOS, IFROS) of an IFTS (X, τ) is denoted by IFO(X) (respectively IFSO(X), IFαO(X), IFRO(X)).

Definition 2.6:[7] An IFS $A = \langle x, \mu_A, \nu_A \rangle$ in an IFTS (X, τ) is said to be an

(i) intuitionistic fuzzy semi closed set (IFSCS in short) if $int(cl(A)) \subseteq A$,

(ii) intuitionistic fuzzy α-closed set (IFαCS in short) if $cl(int(cl(A)) \subseteq A$,

(iii) intuitionistic fuzzy regular closed set (IFRCS in short) if $A = cl(int((A)$.

The family of all IFCS (respectively IFSCS, IFαCS, IFRCS) of an IFTS (X, τ) is denoted by IFC(X) (respectively IFSC(X), IFαC(X), IFRC(X)).

Definition 2.7:[9] An IFS A of an IFTS (X, τ) is an

(i) intuitionistic fuzzy pre closed set (IFPCS in short) if $cl(int(A)) \subseteq A$,

(ii) intuitionistic fuzzy pre open set (IFPOS in short) if $A \subseteq int(cl(A))$.

Definition 2.8:[3] Let A be an IFS in an IFTS (X, τ). Then

$sint(A) = \cup \{ G / G$ is an IFSOS in X and $G \subseteq A \}$,

$scl(A) = \cap \{ K / K$ is an IFSCS in X and $A \subseteq K \}$.

Definition 2.9:[10] An IFS A in an IFTS (X, τ) is an

(i) intuitionistic fuzzy generalized closed set (IFGCS in short) if $cl(A) \subseteq U$ whenever $A \subseteq U$ and U is an IFOS in X.

(ii) intuitionistic fuzzy regular generalized closed set (IFRGCS in short) if $cl(A) \subseteq U$ whenever $A \subseteq U$ and U is an IFROS in X.

Definition 2.10:[10] An IFS A in an IFTS (X, τ) is said to be an intuitionistic fuzzy generalized semi closed set (IFGSCS in short) if $scl(A) \subseteq U$ whenever $A \subseteq U$ and U is an IFOS in (X, τ).

Definition 2.11:[10] An IFS A is said to be an intuitionistic fuzzy generalized semi open set (IFGSOS in short) in X if the complement A^c is an IFGSCS in X.

Definition 2.12:[10] Let f be a mapping from an IFTS (X, τ) into an IFTS (Y, σ). Then f is said to be intuitionistic fuzzy closed map (IF closed map in short) if $f(A) \in IFC(Y)$ for every IFCS A in X .

Definition 2.13:[10] Let f be a mapping from an IFTS (X, τ) into an IFTS (Y, σ). Then f is said to be

(i) intuitionistic fuzzy α- closed map (IFα closed map in short) if $f(A) \in IF\alpha C(Y)$ for every IFCS A in X .
(ii) intuitionistic fuzzy pre closed map (IFP closed map in short) if $f(A) \in IFPC(Y)$ for every IFCS A in X .
(iii) intuitionistic fuzzy generalized closed map (IFG closed map in short) if $f(A) \in IFGCS(Y)$ for every IFCS A in X.

Definition 2.14:[8] Let f be a mapping from an IFTS (X, τ) into an IFTS (Y, σ). Then f is said to be an intuitionistic fuzzy almost closed (IFA closed in short) mapping if f(A) is an IFCS in Y for every IFRCS A in X.

Result 2.15:[10] Every IF closed map is an IFG closed map but the converse may not be true in general.

Definition 2.16:[10]A mapping $f: (X, \tau) \to (Y,\sigma)$ is called an *intuitionistic fuzzy generalized semi closed map* (IFGS closed map in short) if f (A) is an IFGSCS in (Y, τ) for every IFCS A of (X, τ).

Definition 2.17: [9] An IFTS (X, τ) is said to be an intuitionistic fuzzy $\hat{\beta}aT_{1/2}$ (IF$\hat{\beta}aT_{1/2}$ in short) space if every IF$\hat{\beta}$GCS in X is an IFCS in X.

Definition 2.18: [9] An IFTS (X, τ) is said to be an intuitionistic fuzzy $\hat{\beta}_bT_{1/2}$ (IF$\hat{\beta}_bT_{1/2}$ in short) space if every IF$\hat{\beta}$GCS in X is an IFGCS in X.

Definition 2.19: [9] An IFTS (X, τ) is said to be an intuitionistic fuzzy $\hat{\beta}_cT_{1/2}$ (IF$\hat{\beta}_cT_{1/2}$ in short) space if every IF$\hat{\beta}$GCS in X is an IFGSCS in X.

3. Intuitionistic fuzzy $\widehat{\beta}$ generalized closed mappings

In this section, we have introduced intuitionistic fuzzy $\widehat{\beta}$ generalized closed mappings and studied some of their properties.

Definition 3.1: A mapping $f : (X, \tau) \to (Y,\sigma)$ is called an intuitionistic fuzzy $\widehat{\beta}$ generalized closed (IF$\widehat{\beta}$G closed in short) mapping if for every IFCS A of (X, τ), f(A) is an IF$\widehat{\beta}$GCS in (Y, σ).

Example 3.2: Let X = { a, b }, Y = { u, v } and $G_1 = \langle$ x, (0.5, 0.6), (0.3, 0.1) $\rangle$, $G_2 = \langle$ y, (0.1, 0.1), (0.5, 0.6) $\rangle$. Then $\tau = \{ 0_{\sim}, G_1, 1_{\sim} \}$ and $\sigma = \{ 0_{\sim}, G_2, 1_{\sim} \}$ are IFTs on X and Y respectively. Define a mapping $f : (X, \tau) \to (Y, \sigma)$ by f(a) = u and f(b) = v. Then f is an IF$\widehat{\beta}$G closed mapping.

Definition 3.3: A mapping $f : (X, \tau) \to (Y, \sigma)$ is called an intuitionistic fuzzy $\widehat{\beta}$ generalized open (IF$\widehat{\beta}$G open in short) mapping if for every IFOS A of (X, τ), f(A) is an IF$\widehat{\beta}$GOS in (Y, σ).

Example 3.4: Let X = { a, b }, Y = { u, v } and $G_1 = \langle$ x, (0.5, 0.1), (0.2, 0.1) $\rangle$, $G_2 = \langle$ y, (0.4, 0.5), (0.4, 0.3) $\rangle$. Then $\tau = \{ 0_{\sim}, G_1, 1_{\sim} \}$ and $\sigma = \{ 0_{\sim}, G_2, 1_{\sim} \}$ are IFTs on X and Y respectively. Define a mapping $f : (X, \tau) \to (Y, \sigma)$ by f(a) = u and f(b) = v. Then f is an IF$\widehat{\beta}$G open mapping.

Theorem 3.5: Every IF closed mapping is an IF$\widehat{\beta}$G closed mapping but not conversely.

Proof: Let $f : (X, \tau) \to (Y, \sigma)$ be an IF closed mapping. Let A be an IFCS in X. Since f is an IF closed mapping, f(A) is an IFCS in Y. Since every IFCS is an IF$\widehat{\beta}$GCS, f(A) is an IF$\alpha\widehat{\beta}$GCS in Y. Hence f is an IF$\widehat{\beta}$G closed mapping.

Example 3.6: Let X = { a, b }, Y = { u, v } and $G_1 = \langle$ x, (0.3, 0.1), (0.4, 0.5) $\rangle$, $G_2 = \langle$ y, (0.4, 0.3), (0.4, 0.5) $\rangle$. Then $\tau = \{ 0_{\sim}, G_1, 1_{\sim} \}$ and $\sigma = \{ 0_{\sim}, G_2, 1_{\sim} \}$ are IFTs on X and Y respectively. Define a mapping $f : (X, \tau) \to (Y, \sigma)$ by f(a) = u and f(b) = v. Then f is an IF$\widehat{\beta}$G closed mapping. But f is not an IF closed mapping since $G_1^c = \langle$ x, (0.4, 0.5), (0.3, 0.1) $\rangle$ is an IFCS in X but $f(G_1^c) = \langle$ y, (0.4, 0.5), (0.3, 0.1) $\rangle$ is not an IFCS in Y.

Theorem 3.7: Every IFG closed mapping is an IF$\widehat{\beta}$G closed mapping but not conversely.

Proof: Let $f : (X, \tau) \to (Y, \sigma)$ be an IFG closed mapping. Let A be an IFCS in X. Then by hypothesis, f(A) is an IFGCS in Y. Since every IFGCS is an IF$\widehat{\beta}$GCS, f(A) is an IF$\widehat{\beta}$GCS in Y. Hence f is an IF$\widehat{\beta}$G closed mapping.

Example 3.8: Let X = { a, b }, Y = { u, v } and $G_1 = \langle$ x, (0.3, 0.8), (0.1, 0) $\rangle$, $G_2 = \langle$ y, (0.1, 0.7), (0.2, 0.1) $\rangle$. Then $\tau = \{ 0_{\sim}, G_1, 1_{\sim} \}$ and $\sigma = \{ 0_{\sim}, G_2, 1_{\sim} \}$ are IFTs on X and Y respectively. Define a mapping $f : (X, \tau) \to (Y, \sigma)$ by f(a) = u and f(b) = v. Then f is an IF$\widehat{\beta}$G closed mapping. But f is not an IFG closed mapping, since $G_1^c = \langle$ x, (0.1, 0), (0.3, 0.8) $\rangle$ is an IFCS in X but $f(G_1^c) = \langle$ y, (0.1, 0), (0.3, 0.8) $\rangle$ is not an IFGCS in Y.

Theorem 3.9: Every IFα closed mapping is an IF$\widehat{\beta}$G closed mapping but not conversely.

Proof: Let $f : (X, \tau) \to (Y,\sigma)$ be an IFα closed mapping. Let A be an IFCS in X. Then by hypothesis, f(A) is an IFαCS in Y. Since every IFαCS is an IF$\widehat{\beta}$GCS, f(A) is an IF$\widehat{\beta}$GCS in Y. Hence f is an IF$\widehat{\beta}$G closed mapping.

Example 3.10: Let X = { a, b }, Y = { u, v }, $G_1 = \langle x, (0.4, 0.5), (0.3, 0.3) \rangle$ $G_2 = \langle y, (0.3, 0.1), (0.5, 0.6) \rangle$ and $G_3 = \langle y, (0.7, 0.7), (0.1, 0.1) \rangle$. Then $\tau = \{ 0_{\sim}, G_1, 1_{\sim} \}$ and $\sigma = \{ 0_{\sim}, G_2, G_3, 1_{\sim} \}$ are IFTs on X and Y respectively. Define a mapping $f : (X, \tau) \to (Y, \sigma)$ by f(a) = u and f(b) = v. Then f is an IF$\widehat{\beta}$G closed mapping. But f is not an IFα closed mapping, since $G_1^c = \langle x, (0.3, 0.3), (0.4, 0.5) \rangle$ is an IFCS in X but $f(G_1^c) = \langle y, (0.3, 0.3), (0.4, 0.5) \rangle$ is not an IFαCS in Y.

Remark 3.11: IFP closed mapping and IF$\widehat{\beta}$G closed mapping are independent of each other.

Example 3.12: Let X = { a, b }, Y = { u, v } and $G_1 = \langle x, (0.4, 0.1), (0.3, 0.2) \rangle$, $G_2 = \langle y, (0.3, 0.5), (0.1, 0.1) \rangle$, Then $\tau = \{ 0_{\sim}, G_1, 1_{\sim} \}$ and $\sigma = \{ 0_{\sim}, G_2, 1_{\sim} \}$ are IFTs on X and Y respectively. Define a mapping $f : (X, \tau) \to (Y, \sigma)$ by f(a) = u and f(b) = v. Then f is an IFP closed mapping. But f is not an IF$\widehat{\beta}$G closed mapping since $G_1^c = \langle x, (0.3, 0.2), (0.4, 0.1) \rangle$ is an IFCS in X but $f(G_1^c) = \langle y, (0.3, 0.2), (0.4, 0.1) \rangle$ is not an IF$\widehat{\beta}$GCS in Y.

Example 3.13: Let X = { a, b }, Y = { u, v } and $G_1 = \langle x, (0.2, 0.1), (0.6, 0.6) \rangle$, $G_2 = \langle y, (0.5, 0.1), (0.5, 0.8) \rangle$. Then $\tau = \{ 0_{\sim}, G_1, 1_{\sim} \}$ and $\sigma = \{ 0_{\sim}, G_2, 1_{\sim} \}$ are IFTs on X and Y respectively. Define a mapping $f : (X, \tau) \to (Y, \sigma)$ by f(a) = u and f(b) = v. Then f is an IF$\widehat{\beta}$G closed mapping. But f is not an IFP closed mapping since $G_1^c = \langle x, (0.6, 0.6), (0.2, 0.1) \rangle$ is an IFCS in X but $f(G_1^c) = \langle y, (0.6, 0.6), (0.2, 0.1) \rangle$ is not an IFPCS in Y.

Remark 3.14: IFA closed mapping and IF$\widehat{\beta}$G closed mapping are independent of each other.

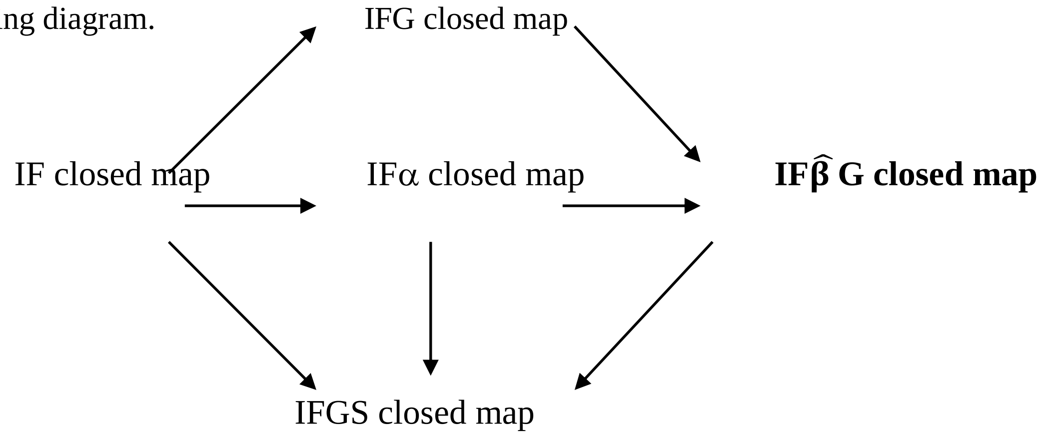
Example 3.15: Let X = { a, b }, Y = { u, v } and $G_1 = \langle x, (0.5, 0.2), (0.4, 0.3) \rangle$, $G_2 = \langle y, (0.4, 0.6), (0.2, 0.2) \rangle$, Then $\tau = \{ 0_{\sim}, G_1, 1_{\sim} \}$ and $\sigma = \{ 0_{\sim}, G_2, 1_{\sim} \}$ are IFTs on X and Y respectively. Define a mapping $f : (X, \tau) \to (Y, \sigma)$ by f(a) = u and f(b) = v. Then f is an IFA closed mapping. But f is not an IF$\widehat{\beta}$G closed mapping since $G_1^c = \langle x, (0.4, 0.3), (0.5, 0.2) \rangle$ is an IFCS in X but $f(G_1^c) = \langle y, (0.4, 0.3), (0.5, 0.2) \rangle$ is not an IF$\widehat{\beta}$GCS in Y.

Example 3.16: Let X = { a, b }, Y = { u, v } and $G_1 = \langle x, (0.3, 0.2), (0.4, 0.5) \rangle$, $G_2 = \langle y, (0.3, 0.4), (0.6, 0.5) \rangle$. Then $\tau = \{ 0_{\sim}, G_1, 1_{\sim} \}$ and $\sigma = \{ 0_{\sim}, G_2, 1_{\sim} \}$ are IFTs on X and Y respectively. Define a mapping $f : (X, \tau) \to (Y, \sigma)$ by f(a) = u and f(b) = v. Then f is an IF$\widehat{\beta}$G closed mapping but not an IFA closed mapping since $G_1^c = \langle x, (0.4, 0.5), (0.3, 0.2) \rangle$ is an IFRCS in X but $f(G_1^c) = \langle y, (0.4, 0.5), (0.3, 0.2) \rangle$ is not an IFCS in Y.

Theorem 3.17: Let $f : (X, \tau) \to (Y,\sigma)$ be a mapping and let f(A) be an IFRCS in Y for every IFCS A in X. Then f is an IF$\widehat{\beta}$G closed mapping.

Proof: Let A be an IFCS in X. Then f(A) is an IFRCS in Y. Since every IFRCS is an IF$\widehat{\beta}$GCS, f(A) is an IF$\widehat{\beta}$GCS in Y. Hence f is an IF$\widehat{\beta}$G closed mapping.

The relations between various types of intuitionistic fuzzy closed mapping are given in the following diagram.



The reverse implications are not true in general.

Theorem 3.18: Let f : (X, τ) → (Y, σ) be an IF$\widehat{\beta}$G closed mapping. Then f is an IF closed mapping if Y is an IF$\widehat{\beta}_a T_{1/2}$ space.

Proof: Let A be an IFCS in X. Then f(A) is an IF$\widehat{\beta}$GCS in Y, by hypothesis. Since Y is an IF$\widehat{\beta}_a T_{1/2}$ space, f(A) is an IFCS in Y. Hence f is an IF closed mapping.

Theorem 3.19: Let f : (X, τ) → (Y, σ) be a mapping from an IFTS X into an IFTS Y. Then the following conditions are equivalent if Y is an IF$\widehat{\beta}_a T_{1/2}$ space:

(i) f is an IF$\widehat{\beta}$G open mapping
(ii) If A is an IFOS in X then f(A) is an IF$\widehat{\beta}$GOS in Y
(iii) f(int(A)) ⊆ int(cl(int(f(A))) for every IFS A in X.

Proof: (i) ⇒ (ii): It is obviously true.

(ii) ⇒ (iii): Let A be any IFS in X. Then int(A) is an IFOS in X. Then f(int(A)) is an IF$\widehat{\beta}$GOS in Y. Since Y is an IF$\widehat{\beta}_a T_{1/2}$ space, f(int(A)) is an IFOS in Y. Therefore, f(int(A)) = int(f(int(A))) ⊆ int(cl(int(f(A)))).

(iii) ⇒ (i): Let A be an IFCS in X. Then its complement A^c is an IFOS in X. By hypothesis, f(int(A^c)) ⊆ int(cl(int(f(A^c)))). This implies f(A^c) ⊆ int(cl(int(f(A^c)))). Hence f(A^c) is an IFαOS in Y. Since every IFαOS is an IF$\widehat{\beta}$GOS, f(A^c) is an IF$\widehat{\beta}$GOS in X. Therefore f(A) is an IF$\widehat{\beta}$GCS in X. Hence f is an IF$\widehat{\beta}$G closed mapping.

Theorem 3.20: Let f : (X, τ) → (Y, σ) be an IF$\widehat{\beta}$G closed mapping. Then f is an IFG closed mapping if Y is an IF$\widehat{\beta}_b T_{1/2}$ space.

Proof: Let A be an IFCS in X. Then f(A) is an IF$\widehat{\beta}$GCS in Y, by hypothesis. Since Y is an IF$\widehat{\beta}_b T_{1/2}$ space, f(A) is an IFGCS in Y. Hence f is an IFG closed mapping.

Theorem 3.21: Let f : (X, τ) → (Y, σ) be an IF closed mapping and g : (Y, σ) → (Z, η) is an IF$\widehat{\beta}$G closed mapping. Then g ∘ f : (X, τ) → (Z, η) is an IF$\widehat{\beta}$G closed mapping.

Proof: Let A be an IFCS in X. Then f(A) is an IFCS in Y, by hypothesis. Since g is an IF$\widehat{\beta}$G closed mapping, g(f(A)) is an IF$\widehat{\beta}$GCS in Z. Hence g $_{o}$f is an IF$\widehat{\beta}$G closed mapping.

Theorem 3.22: If f : (X, τ) → (Y, σ) be an IF$\widehat{\beta}$G closed mapping and Y is an IF$\widehat{\beta}_{c}T_{1/2}$ space, then f is an IFGS closed mapping.

Proof: Let f : (X, τ) → (Y, σ) be a mapping and let A be an IFCS in X. Then by hypothesis f(A) is an IF$\widehat{\beta}$GCS in Y. Since Y is an IF$\widehat{\beta}_{c}T_{1/2}$ space, f(A) is an IFGSCS in Y. This implies f is an IFGS closed mapping.

Theorem 3.23: Let f : (X, τ) → (Y, σ) be a mapping. Then the following conditions are equivalent if Y is an IF$\widehat{\beta}_{a}T_{1/2}$ space:

(i) f is an IF$\widehat{\beta}$G closed mapping

(ii) cl(int(cl(f(A)))) ⊆ f(cl(A)) for every IFS A in X.

Proof: **(i) ⇒ (ii):** Let A be an IFS in X. Then cl(A) is an IFCS in X. By hypothesis, f(cl(A)) is an IF$\widehat{\beta}$GCS in Y. Since Y is an IF$\widehat{\beta}_{a}T_{1/2}$ space, f(cl(A)) is an IFCS in Y. Therefore, cl(f(cl(A))) = f(cl(A)). Now clearly cl(int(cl(f(A)))) ⊆ cl(f(cl(A))) = f(cl(A)). Hence cl(int(cl(f(A)))) ⊆ f(cl(A)).

(ii) ⇒ (i): Let A be an IFCS in X. By hypothesis cl(int(cl(f(A)))) ⊆ f(cl(A)) = f(A). This implies f(A) is an IFαCS in Y and hence f(A) is an IF$\widehat{\beta}$GCS in Y. That is f is an IF$\widehat{\beta}$G closed mapping.

Definition 3.24: A mapping f : X → Y is said to be an intuitionistic fuzzy i-$\widehat{\beta}$ generalized closed mapping (IFi$\widehat{\beta}$G closed mapping in short) if f(A) is an IF$\widehat{\beta}$GCS in Y for every IF$\widehat{\beta}$GCS A in X.

Example 3.25: Let X = { a, b }, Y = { u, v } and G_1 = ⟨ x, (0.1, 0.2), (0.7, 0.6) ⟩, G_2 = ⟨ y, (0.2, 0.2), (0.6, 0.6) ⟩. Then τ = { $0_\sim$, G_1, $1_\sim$ } and σ = { $0_\sim$, G_2, $1_\sim$ } are IFTs on X and Y respectively. Define a mapping f : (X, τ) → (Y, σ) by f(a) = u and f(b) = v. Then f is an IFi$\widehat{\beta}$G closed mapping.

Theorem 3.26: Every IFi$\widehat{\beta}$G closed mapping is an IF$\widehat{\beta}$G closed mapping but not conversely.

Proof: Assume that the mapping f : X → Y be an IFi$\widehat{\beta}$G closed mapping. Let A be an IFCS in X. Then A is an IF$\widehat{\beta}$GCS in X. By hypothesis, f(A) is an IF$\widehat{\beta}$GCS in Y. Hence f is an IF$\widehat{\beta}$G closed mapping.

Example 3.27: Let X = { a, b }, Y = { u, v } and G_1 = ⟨ x, (0.6, 0.6), (0.2, 0.1) ⟩, G_2 = ⟨ y, (0.4, 0.2), (0.4, 0.7) ⟩. Then τ = { $0_\sim$, G_1, $1_\sim$ } and σ = { $0_\sim$, G_2, $1_\sim$ } are IFTs on X and Y respectively. Define a mapping f : (X, τ) → (Y, σ) by f(a) = u and f(b) = v. Then f is an IF$\widehat{\beta}$G closed mapping. But f is not an IFi$\widehat{\beta}$G closed mapping since A = ⟨ x, (0.4, 0.1), (0.6, 0.8) ⟩ is an IF$\widehat{\beta}$GCS in X but f(A) = ⟨ y, (0.4, 0.1), (0.6, 0.8) ⟩ is not an IF$\widehat{\beta}$GCS in Y.

Theorem 3.28: If f : X $\rightarrow$ Y be a bijective mapping then the following are equivalent:

(i) f is an IFi$\widehat{\beta}$G closed mapping

(ii) f(A) is an IF$\widehat{\beta}$GCS in Y for every IF$\widehat{\beta}$GCS A in X

(iii) f(A) is an IF$\widehat{\beta}$GOS in Y for every IF$\widehat{\beta}$GOS A in X

Proof: (i) $\Leftrightarrow$ (ii): It is obviously true.

(ii) $\Rightarrow$ (iii): Let A be an IF$\widehat{\beta}$GOS in X Then A^c is an IF$\widehat{\beta}$GCS in X. By hypothesis, f(A^c) is an IF$\widehat{\beta}$GCS in Y. That is $f(A)^c$ is an IF$\widehat{\beta}$GCS in Y. Hence f(A) is an IF$\widehat{\beta}$GOS in Y.

(iii) $\Rightarrow$ (i): Let A be an IF$\widehat{\beta}$GCS in X. Then A^c is an IF$\widehat{\beta}$GOS in X. By hypothesis, f(A^c) is an IF$\widehat{\beta}$GOS in Y. That is $f(A)^c$ is an IF$\widehat{\beta}$GOS in Y. Hence f(A) is an IF$\widehat{\beta}$GCS in Y. Thus f is an IFi$\widehat{\beta}$G closed mapping.

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