Skolem Difference Mean Labeling of Some Path Related Graphs

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Abstract

A graph $G = (V(G), E(G))$ with p vertices and q edges is called Skolem difference mean labeling graph if $f: V \rightarrow \{1,2,3,. . ., p+q\}$ is an injective mapping such that induced bijective edge labeling $f^*: E \rightarrow \{1,2,3,. . ., q\}$ defined by $f^*(e = uv) = \frac{|f(u)-f(v)|}{2}$, if $|f(u) - f(v)|$ is even otherwise $\frac{|f(u)-f(v)|+1}{2}$, if $|f(u) - f(v)|$ is odd.. In this paper we discuss Skolem difference mean labeling of Broom graph, Comb graph, Two star graph and $K_{1,3} * K_{1,n}$ graph.

Keywords: *Skolem difference mean labeling, Broom graph, Comb graph, Two star graph, $K_{1,3} * K_{1,n}$ graph.*

1. Introduction

We consider finite, connected and undirected graph. We consider graph G having set of vertices $V(G)$ and set of edges $E(G)$. A graph labeling of a graph is a map that carries the graph elements to the set of numbers, subject to certain conditions. For detailed survey on graph labeling we refer to a dynamic survey on graph labeling by Gallian [2].

The concept of Skolem mean labeling was introduced by V. Balaji, D.S.T. Ramesh and A. Subramanian in [7]. Motivated by these definition Skolem difference mean labeling was introduced by K. Murugan and A. Subramanian in [4]. For the standard notation, we refer Gross and Yellen [3].

Definition 1.1 A graph $G = (V(G), E(G))$ with p vertices and q edges is called Skolem difference mean labeling graph if $f: V \rightarrow \{1,2,3,. . ., p+q\}$ is an injective mapping such that induced bijective edge labeling $f^*: E \rightarrow \{1,2,3,. . ., q\}$ defined by $f^*(e = uv) = \frac{|f(u)-f(v)|}{2}$, if $|f(u) - f(v)|$ is even otherwise $\frac{|f(u)-f(v)|+1}{2}$, if $|f(u) - f(v)|$ is odd. A graph that admits Skolem difference mean labeling is called Skolem difference mean graph

Definition 1.2 A Broom graph is graph of n vertices and have a path P with d vertices and $n-d$ pendant vertices all of these being adjacent to either the origin u or the terminus v of path P. It is denoted by $B_{n,d}$.

Definition 1.3 The Comb is the graph obtained from a path P_n by attaching a pendant vertex to each vertex of the path. It is denoted by $P_n \odot K_1$.

Definition 1.4 The two star is the disjoint union of $K_{1,m}$ & $K_{1,n}$. It is denoted by $K_{1,m} \cup K_{1,n}$.

Definition 1.5 $K_{1,3} * K_{1,n}$ is the graph obtained from $K_{1,3}$ by attaching root of a star $K_{1,n}$ at each pendant vertex of $K_{1,3}$.

2. Main Result

Theorem 2.1 The Broom graph $B_{n,d}$ is a Skolem difference mean graph for all values of $n \geq 4, d \geq 2$.

Proof: Let $G = B_{n,d}$ with $V(G) = \{u_1, u_2, \dots\dots, u_d, u_{d+1}, \dots\dots\dots, u_n\}$ and

$E(G) = \{(u_i\, u_{i+1}) : 1 \leq i \leq d-1\} \cup \{(u_d\, u_{d+i}) : 1 \leq i \leq \text{n} - d\}$.

Hence $|V(G)| = n$ & $|E(G)| = n - 1$.

Define a function $f: V(G) \to \{1,2,3, \dots\dots\dots\dots.., 2n-1\}$ as follows.

Case:1 When d is even.

$$f(u_{2i-1}) = d + 2i - 1 \qquad 1 \leq i \leq \frac{d}{2}$$

$$f(u_{2i}) = d - 2i + 1 \qquad 1 \leq i \leq \frac{d}{2}$$

$$f(u_i) = 2i - 1 \qquad d + 1 \leq i \leq n.$$

Case:2 When d is odd.

$$f(u_{2i-1}) = d - 2i + 2 \qquad 1 \leq i \leq \frac{d+1}{2}$$

$$f(u_{2i}) = d + 2i \qquad 1 \leq i \leq \frac{d-1}{2}$$

$$f(u_i) = 2i - 1 \qquad d + 1 \leq i \leq n.$$

In both case we define following edge function

$f^*: E(G) \to \{1,2,3 \dots, n-1\}$ by $f^*(u_i u_{i+1}) = i \qquad 1 \leq i \leq d-1$

$$f^*(u_d, u_{i+1}) = i \qquad d \leq i \leq n$$

Which is bijective function. Hence Broom graph is a Skolem difference mean graph.

Example-2.2: A Skolem difference mean labeling of $B_{15,10}$ is shown in Figure 1.

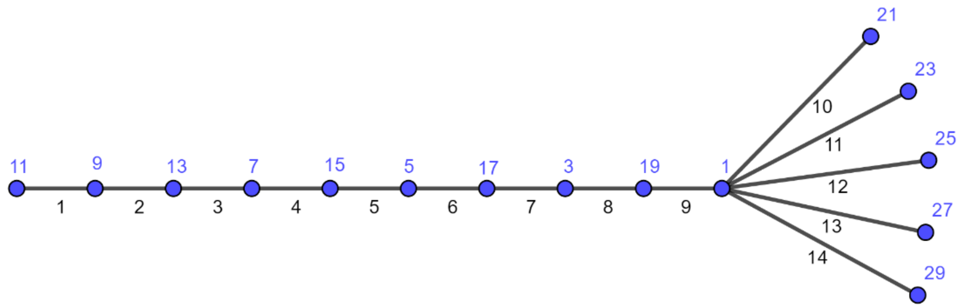

Figure 1. Skolem difference mean labeling of $B_{15,10}$

Theorem 2.3 The Comb $P_n \odot K_1$ graph is a Skolem difference mean graph for all $n \geq 2$.

Proof: Let $G = P_n \odot K_1$ with $V(G) = \{u_i, v_i : 1 \leq i \leq n\}$ and

$E(G) = \{(u_i\, u_{i+1}) : 1 \leq i \leq n-1\} \cup \{(u_i\, v_i) : 1 \leq i \leq n\}$.

$\therefore$ $|V(G)| = 2n$ & $|E(G)| = 2n - 1$.

Define $f : V(G) \to \{1,2,3, \dots \dots \dots \dots \dots \dots ., 4n-1\}$ as follows.

Case- 1 If n is odd

$$f(u_{2i-1}) = 4n - 2i \qquad 1 \leq i \leq \frac{n+1}{2}$$

$$f(u_{2i}) = 2i - 1 \qquad 1 \leq i \leq \frac{n-1}{2}$$

$$f(v_{2i-1}) = 4n + 3 - 6i \qquad 1 \leq i \leq \frac{n+1}{2}$$

$$f(v_{2i}) = 6i - 2 \qquad 1 \leq i \leq \frac{n-1}{2}$$

Case -2 If n is even

$$f(u_{2i-1}) = 4n - 2i \qquad 1 \leq i \leq \frac{n}{2}$$

$$f(u_{2i}) = 2i - 1 \qquad 1 \leq i \leq \frac{n}{2}$$

$$f(v_{2i-1}) = 4n + 3 - 6i \qquad 1 \leq i \leq \frac{n}{2}$$

$$f(v_{2i}) = 6i - 2 \qquad 1 \leq i \leq \frac{n}{2}$$

In both case we define following edge function

$f^* : E(G) \to \{1,2,3 \dots, 2n-1\}$ by $f^*(u_i u_{i+1}) = 2n - i \qquad 1 \leq i \leq n-1$

$$f^*(u_i v_i) = i \qquad 1 \leq i \leq n$$

Which is bijective function. Hence Comb $P_n \odot K_1$ graph is a Skolem difference mean graph.

Example-2.4: A Skolem difference mean labeling of $P_8 \odot K_1$ is shown in Figure 2.

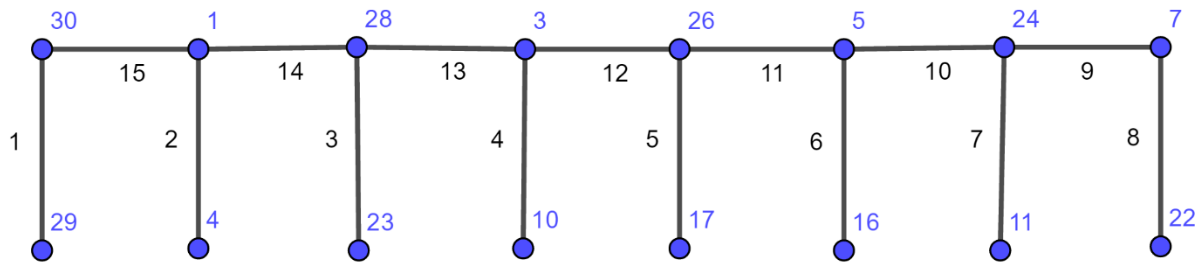

Figure 2. Skolem difference mean labeling of $P_8 \odot K_1$

Theorem 2.5 The two star $K_{1,m} \cup K_{1,n}$ is a Skolem difference mean graph for all values of $m, n \geq 2$.

Proof: Let $G = K_{1,m} \cup K_{1,n}$

Case 1: $n = m$.

We have $V(G) = \{u, v\} \cup \{u_{i,}, v_i / 1 \leq i \leq m\}$.

Hence $|V(G)| = 2m + 2$ and $|E(G)| = 2m$.

The required vertex labeling $f: V(G) \rightarrow \{1,2,3 \dots \dots \dots \dots \dots., 4m + 2\}$ is defined as follows:

$f(u) = 1$

$f(v) = 3$

$f(u_i) = 2i \qquad ,1 \leq i \leq m$

$f(v_i) = 2m + 2i + 2, \; 1 \leq i \leq m.$

We define following edge function

$f^*: E(G) \rightarrow \{1,2,3, \dots \dots \dots \dots \dots \dots, 2m\}$ by

$f^*(uu_i) = i \qquad 1 \leq i \leq m$

$f^*(vv_i) = m + i \quad 1 \leq i \leq m$

Thus induced edge labels are distinct from $1,2,3, \dots \dots \dots \dots., 2m$.

Case 2: $n < m \; or \; m < n$.

We have $V(G) = \{u, v\} \cup \{u_{i,}, v_j / 1 \leq i \leq m, 1 \leq j \leq n\}$.

$|V(G)| = m + n + 2$

$|E(G)| = m + n.$

$f: V(G) \to \{1,2,3 \ldots \ldots \ldots \ldots \ldots, 2m + 2n + 2\}$ is defined as follows:

$f(u) = 1$

$f(v) = 3$

$f(u_i) = 2i \quad , 1 \leq i \leq m$

$f(v_j) = 2m + 2j + 2, \ 1 \leq j \leq n.$

We define following edge function

$f^*: E(G) \to \{1,2,3, \ldots \ldots \ldots \ldots \ldots \ldots, m + n\}$ by

$f^*(uu_i) = i \quad 1 \leq i \leq m$

$f^*(vv_j) = m + j \quad 1 \leq j \leq n$

Thus the induced edge labels are distinct from $1,2,3, \ldots \ldots \ldots \ldots .., m + n$.

Hence, the Star graph is a Skolem difference mean graph.

Example-2.6: A Skolem difference mean labeling of $K_{1,7} \cup K_{1,8}$ is shown in Figure 3.

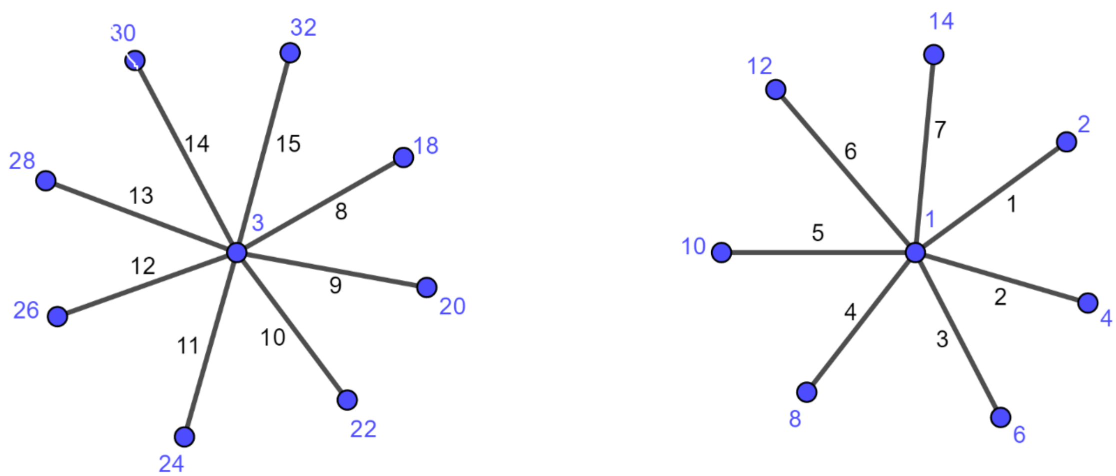

Figure 3. Skolem difference mean labeling of $K_{1,7} \cup K_{1,8}$

Theorem 2.7 The graph $K_{1,3} * K_{1,n}$ is a Skolem difference mean graph for all $n \geq 2$.

Proof: Let $G = K_{1,3} * K_{1,n}$ with $V(G) = \{x, u, v, w, u_i, v_i, w_i : 1 \leq i \leq n\}$ and $E(G) = \{xu, xv, xw, uu_i, vv_i, ww_i : 1 \leq i \leq n\}$. Hence $|V(G)| = 3n + 4$ and $|E(G)| = 3n + 3$

Define $f: V(G) \to \{1,2,3, \ldots \ldots \ldots \ldots \ldots \ldots, 6n + 7\}$ by

$f(u) = 1$

$f(v) = 3$

$f(w) = 2n + 5$

$f(x) = 4n + 7$

$f(u_i) = 6n + 9 - 2i \qquad 1 \le i \le n$

$f(v_i) = 4n + 7 - 2i \qquad 1 \le i \le n$

$f(w_i) = 2n + 5 - 2i \qquad 1 \le i \le n.$

We define following edge function $f^*: E(G) \to \{1,2,3, \dots\dots\dots\dots\dots, 3n + 3\}$ by

$f^*(uu_i) = 3n - i + 4 \qquad 1 \le i \le n$

$f^*(vv_i) = 2n - i + 2 \qquad 1 \le i \le n$

$f^*(ww_i) = i \qquad 1 \le i \le n$

$f^*(ux) = 2n + 3$

$f^*(xv) = 2n + 2$

$f^*(xw) = n + 1$

Thus the induced edge labels are distinct from $1,2,3, \dots\dots\dots\dots, 3n + 3$.

Hence the graph $K_{1,3} * K_{1,n}$ is a Skolem difference mean graph.

Example-2.8: A Skolem difference mean labeling of $K_{1,3} * K_{1,5}$ is shown in Figure-4.

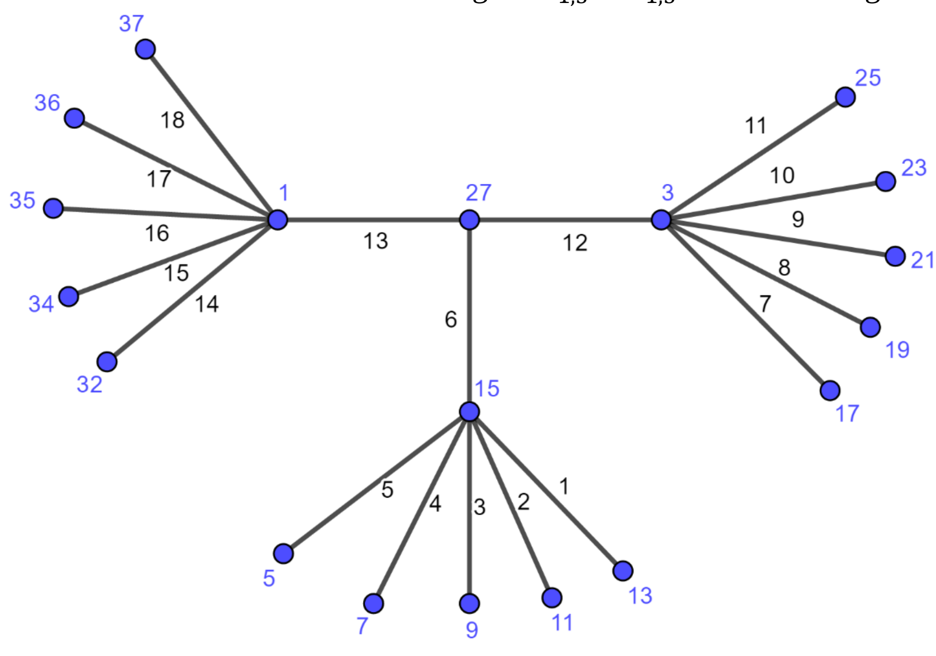

Figure 4. Skolem difference mean labeling of $K_{1,3} * K_{1,5}$

3. Conclusion

In this paper we prove that the graphs $B_{n,d}$, $P_n \odot K_1$, $K_{1,3} * K_{1,n}$, $K_{1,m} \cup K_{1,n}$ admits Skolem difference mean labeling.

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