A Note on Pascal's Triangle and Its Applications

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Abstract

In this paper we have effectively use the Pascal's Triangle and Binomial theorem to find ordinary decimal expansion of the number of the form n^k where n & k are any natural number.

Key Words: *Pascal's Triangle, Square of numbers, Binomial expression.*

1. Introduction:

In elementary school as well as in day to day life many time the problem of finding the ordinary decimal expansion of numbers like $8^{13}, 27^{35}, 234^{47}, etc$ appears. If number is one or two digits or power of number is one digit, then such a calculation is easy using direct method i.e. multiplication of number to itself. But if numbers contains more digits or power of number is large then such a calculation using direct method is time consuming or boring. Also digital calculators have limitations like number of digits.

In this paper we have used Pascal's triangle to find ordinary decimal expansion of numbers of the form n^k where k is positive integer. Such a method is useful in the case n contains more digits or power k is large.

2. Binomial Expression:

Any mathematical expression consisting of two terms connected by positive (+) or negative (−) sign is called as Binomial Expression.

$x + a$, $2x + 7$, $7x - 5y$, $x^2 + 2x$ are the examples of binomial expressions. $(x + y)^n$ is also an example of binomial expression, where n is integer.

We know that,

$$(x + y)^2 = \mathbf{1}\, x^2 + \mathbf{2}\, x\, y + \mathbf{1}\, y^2$$

$$(x + y)^3 = \mathbf{1}\, x^3 + \mathbf{3}\, x^2\, y + \mathbf{3}\, x\, y^2 + \mathbf{1}\, y^3$$

$$(x + y)^4 = \mathbf{1}\, x^4 + \mathbf{4}\, x^3\, y^1 + \mathbf{6}\, x^2 y^2 + \mathbf{4}\, x^1\, y^3 + \mathbf{1}\, y^4$$

These are expansion of binomial expressions. In this expression, the boldface numbers 1, 2, 3, 4, 6, etc are called binomial coefficients of variable x and y. We can use the Pascal's triangle to find the numerical values of binomial coefficients.

3. Pascal's Triangle

One of the most interesting number patterns is Pascal's Triangle (P.T.) named after Blaise Pascal, a famous French Mathematician and Philosopher. It is usually write in pyramid shape. It is useful to calculate quickly the numerical values of binomial coefficients for n^{th} order. First and Last coefficient of each row of Pascal Triangle is **1** and another coefficient of each row is equal to sum of coefficient immediately above it and preceding coefficient in the same line above. Each row increase by 1 term from previous row. This working rule is based on

${}^{n+1}_{r}c = {}^{n}_{r}c + {}^{n}_{r-1}c$.

4. Structure of Pascal Triangle

There are many ways to represent the Pascal's triangle. The simple row structure of a Pascal's triangle is shown in following pattern.

1

1 1

1 2 1

1 3 3 1

1 4 6 4 1

1 5 10 10 5 1

1 6 15 20 15 6 1

⋮

5. Rules to obtain a binomial expression using Pascal Triangle:

Consider an expression $(x + y)^n$:

1) The number of term in the expansion of $(x + y)^n$ is $(n + 1)$.

2) The sum of indices of power x & y in any term is always n.
3) The coefficient of x^n & y^n is always 1.
4) The coefficients of the terms between x^n & y^n are defined by using Pascal Triangle.
5) The power of x is start to n & it is steadily decreasing by 1 up to the power of x become 0 $i.e. (n-n)$ and power of y is start to 0 & it is steadily increasing by 1 and for $(n+1)^{th}$ term power of y become n .

Consider the following examples:

1) $(x+y)^4 = 1\,x^4y^0 + 4\,x^3y^1 + 6\,x^2y^2 + 4\,x^1y^3 + 1x^0y^4$

(Here power is 4, hence consider 5^{th} row in P.T.)

2) $(x+y)^7 = 1\,x^7y^0 + 7\,x^6y^1 + 21\,x^5y^2 + 35\,x^4y^3 + 35\,x^3y^4 + 21\,x^2y^5 + 7\,x^1y^6 + 1x^0y^1$

(Here power is 7, hence consider 8^{th} row in P.T.)

3) $(x+y)^0 = 1\,x^0y^0 = 1$ (Here Power is 0, hence consider 1^{st} row in P.T.)

6. Application to find ordinary decimal expansion of n^k:

In this research article we have use Pascal's triangle & binomial coefficients to find the ordinary decimal expansion of positive power of one or more digit numbers.

6.1.Example 1. Find (74)^7

Solution: In a given two digit number let us consider first (tens) digit x & second (unit) digit is y. Power of number is 7, so consider 8^{th} row of Pascal Triangle. Therefore, 8 terms are involved in the power formula as:

$(74)^7 = (x^7, 7x^6y, 21x^5y^2, 35x^4y^3, 35x^3 3y^4, 21x^2y^5, 7xy^6, y^7)$

$= 7^7, 7\cdot7^6\cdot4, 21\cdot7^5\cdot4^2, 35\cdot7^4\cdot4^3, 35\cdot7^3\cdot4^4, 21\cdot7^2\cdot4^5, 7\cdot7\cdot4^6, 4^7$

$= 823543, 3294172, 5647152, 5378240, 3073280, 1053696, 200704, 16384$

We use the following method called carry forward method, to add the numbers on RHS of above equation as follows:

Remove the last digit 4 from last number 16384 and add the remaining number 1638 to next number 200704 we get 202342. Again remove last digit 2 from resulting number 203242 and add remaining number 20324 with next number 1053696 we get 1073930. Continue the said process till last number, we get

$16384 \quad ----------- \textit{Remove last digit } 4$

$1683 + 200704 = 202342 \quad ----------- \textit{Remove last digit } 2$

$20234 + 1053696 = 1073930 \quad ----------- \textit{Remove last digit } 0$

$107393 + 3073280 = 3180673 \quad ----------- \textit{Remove last digit } 3$

$318067 + 5378240 = 5696307 \quad ----------- \textit{Remove last digit } 7$

$569630 + 5647152 = 6216782 \quad ----------- \textit{Remove last digit } 2$

$621678 + 3294172 = 3915850 \quad ----------- \textit{Remove last digit } 0$

$391585 + 823543 = 1215128$

Then write last number 1215128 with removed digits as 12151280273024

Hence $(\mathbf{74})^{7}$**=**$\mathbf{1, 21, 51, 28, 02, 73, 024}$

6.2.Example 2. Find $(58)^{10}$

Solution: Here power of number is 10, so consider 11th row of Pascals triangle. Then

$(58)^{10} = (x^{10}, 10x^9y^1, 45x^8y^2, 120x^7y^3, 210x^6y^4, 252x^5y^5, 210x^4y^6, 120x^3y^7,$

$45x^2y^8, 8x^1y^9, y^{10})$

$= 5^{10}, 10 \cdot 5^9 \cdot 8^1, 45 \cdot 5^8 \cdot 8^2, 120 \cdot 5^7 \cdot 8^3, 210 \cdot 5^6 \cdot 8^4, 252 \cdot 5^5 \cdot 8^5, 210 \cdot 5^4 8^6, 120 \cdot 5^3 8^7, 45 \cdot 5^2 \cdot 8^8, 10 \cdot 5^1 \cdot 8^9, 8^{10}$

$= 9765625, 156250000, 1125000000, 4800000000, 13440000000, 25804800000,$

$34406400000, 31457280000, 18874368000, 6710886400, 1073741824$

Apply the method discussed in example 6.1 we obtain,

$\therefore (\mathbf{58})^{\mathbf{10}} = \mathbf{4, 30, 80, 42, 06, 89, 94, 05, 824}$

6.3. Example 3. Find $(835)^5$

Solution: In this case we consider x = 83 & y = 5. Power of number is 5 so considers 6th row of Pascals triangle. Then

$$(835)^5 = 83^5\ ,5\cdot 83^4\cdot 5\ ,10\cdot 83^3\cdot 5^5\ ,10\cdot 83^2\cdot 5^3\ ,5\cdot 83\cdot 5^4\ ,5^5$$

$$= 3939040643\ ,1186458025\ ,142946750\ ,8611250\ ,259375\ ,3125$$

Here 83^5, 83^4 and so on calculated as discussed in examples 6.1 & 6.2.

Apply the method discussed in example 6.1 we obtain,

$$\therefore\ (\mathbf{835})^{\mathbf{5}} = \mathbf{40, 59, 12, 45, 50, 21, 275}$$

6.4. Example 4. Find $(4918)^4$.

Solution: Here we have four digit number, so let us consider x = 49 & y = 18.Here power is four then select 5th row in Pascals triangle. Then

$$(4918)^4 = 49^4, 4\cdot 49^3\cdot 18, 6\cdot 49^2\cdot 18^2\ ,4\cdot 49\cdot 18^3\ ,18^4$$

$$= 5764801\ ,8470728\ ,4667544\ ,1143072\ ,104976$$

Apply the method discussed in example 6.1 we obtain,

$$\therefore\ (\mathbf{4948})^{\mathbf{4}} = \mathbf{58, 49, 97, 61, 78, 52, 176}$$

7. Conclusion

In this article we effectively use Pascal's triangle as a powerful tools to find the ordinary decimal expansion of the number n^k, where k is a positive integer.

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