Main Double Inequalities on Chromatic Number Corresponding to Magic and Anti-Magic Graphs for Proper Colourings

[#1] R. Balamurugan, K. Sangeetha[*2]

[#1]*Research Scholar, Department of Mathematics, PRIST University*
[*2]*Assistant Professor, Department of Mathematics, PRIST University*

Abstract

This paper introduces, the concept of proper colourings in vertex magic and anti-magic graphs. Labeling is simply a bijection mapping of a graph with natural number assigned to verties or edges and the graph labeling having weights corresponding to each edge/ vertex are consideration. The graph labeling having weights corresponding to each vertex are similar, then the labeling is said to be magic. The graph labeling having weights corresponding to each vertex are not similar, then the labeling is said to be antimagic. This paper establishes the main double inequalities on chromatic number corresponding to magic and anti-magic graphs.

Keywords— *Vertex Magic Labeling, Regular Graphs, Proper Colourings, Anti-magic graphs.*

I. Introduction

Considering finite, simple and undirected graphs for the analysis. The graph H has vertex set U = U(H) and edge set D = D(H) and ε = |D| and u = |U| are taken [1]. Magic labeling and antimagic graph are considered in this analysis. [4], [3]). If the graphs labeling having numbers having every vertex and its incident edges summed to the same number, then it is said 0to be vertex magic graph [2]). And the number obtained is called magic number. Let r be the degree of the vertex of a regular graph.

II. Main Results

2.1. Colourings in r-regular magic graphs for r ≥ 2

The preliminary definitions for reference with some results are given below.

Definition 2.1

A Bijection f : U(H) ∪ D(H) → {1, 2, 3, . . . , v+ε } is called a vertex magic labeling if there is a constant J, such that vertex weight of every vertex in H is equal to the constant

$K = f(x) + \sum_{y \in N(x)} f(xy)$

Definition 2.2

The chromatic number of a graph 'H' is the minimum number of colours that is needed to colour a graph 'H'. It is represented by Γ(H).

Definition 2.3

If every vertex in a graph 'H' has the same degree r, i.e., σ = Δ = r, then a graph 'H' is called a regular graph of degree r/ r-regular graph.

Theorem 2.1

If 'H' is a vertex magic r-regular graphs then the chromatic number satisfies the double inequality

$\frac{j-1}{v+z} \leq \Gamma(H) \leq \frac{1}{2} mr$

Proof

Case: i

Let B_m be a cycle graph.

Let m be an odd integer.

Let U(H) be the set of all vertices in the cycle graph and D(H) be the set of an edges in the cycle graph.

The cycle graph consists of m vertices and m edges.

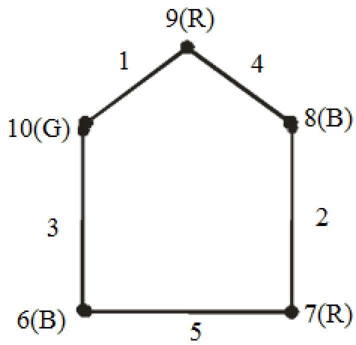
Define $f : U \cup D \rightarrow \{1, 2, 3, \ldots, 2m\}$ as below :

For vertex v_i, the vertex magic labelling is represented for $1 \leq j \leq m$ respectively as below:

$f(v_j) = (2m + 1) - j, \ 1 \leq j \leq m$

For every edge d_j, the magic labeling is represented as below for $1 \leq j \leq m$

$$f(d_j) = \begin{cases} \frac{j+1}{2} & \text{if } j \text{ is odd} \\ \frac{m+j+1}{2} & \text{if } j \text{ is even} \end{cases}$$



Magic Number K=14

Let B_m be a graph with vertex magic labeling, then the vertices v_1, v_2, v_3, v_4, v_5 are coloured with different colours by proper colouring and the number of colours required for colouring the graph is 3.

Therefore, $\Gamma(H) = 3$

Therefore, $\frac{K-1}{v+z} \leq \Gamma(H)$ (1)

The general condition of r-regular graph is represented as $\frac{1}{2} mr$

Therefore, $\Gamma(H) \leq \frac{1}{2} mr$ (2)

From equation (1) and (2), it is easily verified that

$\frac{K-1}{v+z} \leq \Gamma(H) \leq \frac{1}{2} mr$

Thus, an odd cycle satisfies the condition for colourings in vertex magic labeling for 2-regular graphs.

Case: ii

Let C_n be an even cycle

Let m be an even integer

Let $U(B_m)$ be the set of all vertices and $D(B_m)$ be the set of all edges in H.

Let $U(B_m) = \{v_1, v_2, v_3, \ldots, v_n\}$

Let $D(B_m) = \{v_n v_1\} \cup \{v_j v_{j+1}, 1 \leq j \leq m-1\}$

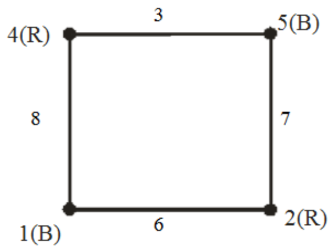
Define $f : V \cup E \rightarrow \{1, 2, 3, \ldots, 2m\}$ as below :

For every vertex v_j, the vertex magic labeling is represented as below.

$$f(v_j) = \begin{cases} j+3 & if\ 1 \le j \le 2 \\ \frac{j+1}{2} & if\ j = 3 \\ j-3 & if\ j = 4 \end{cases}$$

For every edge d_i, the edge labelings is represented as below :

$$f(d_i) = \begin{cases} j+2 & if\ j = 1 \\ 2m - j + 1 & if\ 2 \le j \le 3 \\ 2j & if\ j = 4 \end{cases}$$



Magic Number, K = 15

Let B_m be a cycle graph with vertex magic labeling, then the vertices v_1, v_2, v_3, v_4 are coloured with different colours by proper colouring. The colours used for colouring this graph is 2.

Therefore Γ(H)= 2

$$\therefore \frac{K-1}{v+z} \le \Gamma(H) \qquad (3)$$

The general condition of r-regular graph is denoted as $\frac{1}{2}mr$

Therefore, $\Gamma(H) \le \frac{1}{2}mr \qquad (4)$

From equation (3) and (4), it is easily verified that

$$\frac{K-1}{v+z} \le \Gamma(H) \le \frac{1}{2}mr$$

Thus, an even cycle satisfies the condition for colourings in vertex magic labeling for 2-regular graphs.

Case: iii

Let the graph H be generalized Petersen graph.

Let m be an even integer.

Let $U(B_m) = \{v_1, v_2, v_3, \ldots, v_n\}$

Let $D(B_m) = \{e_1, e_2, e_3, \ldots, e_n\}$

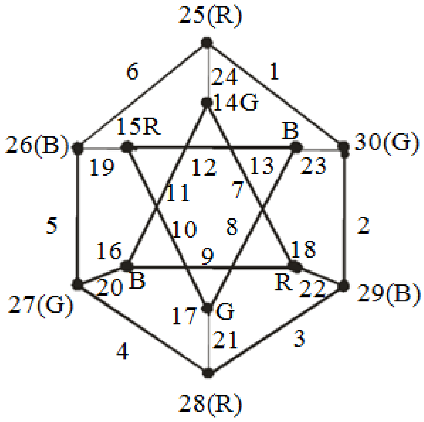
Define $f : V \cup E \to \{1, 2, 3, \ldots, 2m + 6\}$ as below :

For every vertex v_j is represented respectively as below :

$$f(v_j) = \begin{cases} 2m + j, & j = 1 \\ 2m + 8, & 2 \le j \le 6 \\ 2m - j - 3, & 7 \le j \le 8 \\ 2m - j + 3, & 9 \le j \le 12 \end{cases}$$

For every edge d_i, is represented respectively as below :

$$f(d_j) = \begin{cases} j, & 1 \le j \le 12 \\ 3m - j + 1, & 13 \le j \le 18 \end{cases}$$



Magic number, K = 56

The generalized Petersen Graph is labeled with vertex magic, then the vertices are coloured with different colours by proper colouring and the number of colours used for colouring this graph is 3.

Therefore, Γ(H)= 3

$\therefore \frac{K-1}{v+z} \leq \Gamma(H)$

(5)

The general condition of r-regular graph is denoted as $\frac{1}{2}mr$

Therefore, $\Gamma(H) \leq \frac{1}{2}mr$

(6)

From equation (5) and (6), it is easily verified that

$\frac{K-1}{v+z} \leq \Gamma(H) \leq \frac{1}{2}mr$

Thus, a generalized Petersen Graph satisfies the condition for colourings in vertex magic labeling for 3-regular graphs.

Case: iv

Let H be a complete graph

Let M be an odd integer.

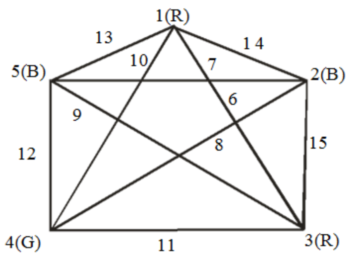
Let U = $\{v_1, v_2, v_3, \ldots, v_m\}$ be the vertices.

Let D = $\{e_1, e_2, e_3, \ldots, e_m\}$ be the edges.

Define $f : V \cup E \rightarrow \{1, 2, 3, \ldots, 3m\}$ as below :

$f(v_j) = j$ for $1 \leq j \leq m$

$f(d_j) = n + j$ for $1 \leq j \leq 2m$



Magic number, K = 45

The complete graphs is labeled with vertex magic, then the vertices in the complete graph is coloured with different colours by proper colouring and the number of colours used for colouring the complete graph is 5.

Therefore, Γ(H)=5

The general condition of r-regular graph is denoted as $\frac{1}{2}mr$

$\therefore \frac{K-1}{v+\varepsilon} \leq \Gamma(H)$

(7)

Therefore, $\Gamma(H) \leq \frac{1}{2}mr$

(8)

From equation (7) and (8), it is easily verified that

$\frac{K-1}{v+\varepsilon} \leq \Gamma(H) \leq \frac{1}{2}mr$

Therefore, the complete graph satisfies the condition for colourings in vertex magic labelings for 4-regular graphs

III. COLOURINGS IN ANTI-MAGIC GRAPHS

The following are some of the basic definitions which are to be referred and has established some results.

Definition 3.1

The weight of a vertex $s \in U$, under the labeling γ, is $wt(x) = \gamma(x) + \sum_{y \in N(x)} \gamma(xy)$, where for every $s \in U$, $\gamma(x) = 0$ under an edge labeling and $\gamma(x) \neq 0$ under a total labeling.

Theorem 3.1

If H is a vertex anti-magic graphs then the chromatic number satisfies double inequality.

$\frac{l_1}{l_0} - 1 \leq \Gamma(H) \leq \frac{a-b'}{m} + b$

Proof :

Case i

Let Q_m be the path and m be an odd integer.

Let $\{v_1, v_2, \ldots, v_n\}$ be the vertices and $\{d_1, d_2, \ldots, d_n\}$ be the edges of the path Q_m receive the following labels.

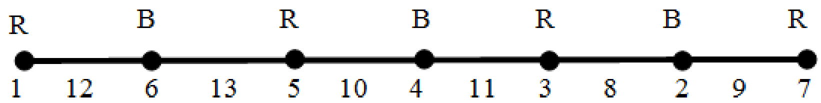
$f(v_1) = 1$

$f(v_m) = m$

$f(v_j) = m-j+1, \; 2 \leq j \leq m-1$

and the edge receive the labels

$$f(v_j, v_{j+1}) = \begin{cases} 2m-1-j, & j \text{ is odd} \\ 2m+1-j, & j \text{ is even} \end{cases}$$



Let Q_m be the path and with vertex antimagic labeling, then the vertices $v_1, v_2, v_3, v_4, v_5, v_6, v_7$ are coloured with different colours by proper colourings and the number of colours used for colouring this graph is 2.

Therefore, $\Gamma(H) = 2$

Let $l_1 = \max\{13, 16, 19, 22, 25, 28, 31\}$

$\therefore l_1 = 31$

Let $l_0 = \min\{13, 16, 19, 22, 25, 28, 31\}$ $\therefore l_0 = 13$

$\frac{l_1}{l_0} - 1 \leq \Gamma(H)$ (1)

$\frac{a-b'}{m} + b \geq \Gamma(H)$

$\therefore \Gamma(H) \leq \frac{a-b'}{m} + b$ (2)

From (1) and (2)

It is easily verified that

$\frac{l_1}{l_0} - 1 \leq \Gamma(H) \leq \frac{a-b'}{m} + b$

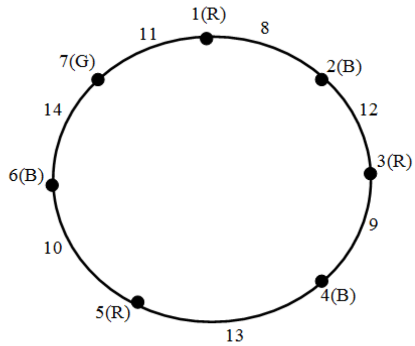
Then the vertex weights form an arithmetic progression with difference two, namely 2m-1,2m+2,2m+3,2m+5, . . .

Case ii

Let B_m be the cycle and m be the number of vertices. Label the vertices and the edges of B_m by

$f(v_j) = m - j + 1$ for $1 \leq i \leq m$

$$f(v_j, v_{j+1}) = \begin{cases} n + \frac{j+1}{2} & \text{if } j \text{ is odd} \\ 2n - 2 + \frac{j}{2} & \text{if } j \text{ is even} \end{cases}$$



Let B_m be the cycle and with vertex antimagic labeling, then the vertices $v_1,v_2,v_3,v_4,v_5,v_6,v_7$ are coloured with different colours by proper colourings and the number of colours used for colouring this graph is 3.

$\therefore \Gamma(H) = 3$

Let $l_1 = \max\{20,22,24,26,28,30,32\}$

$\therefore l_1 = 32$

Let $l_0 = \min\{20,22,24,26,28,30,32\}$

$\therefore t_0 = 20$

$\frac{l_1}{l_0} - 1 \leq \Gamma(H)$

(9)

$\frac{a-b'}{m} + b \geq \Gamma(H)$

$\therefore \Gamma(H) \leq \frac{a-b'}{m} + b$

(10)

From equation (9) and (10), it is easily verified that

$\frac{l_1}{l_0} - 1 \leq \Gamma(H) \leq \frac{a-b'}{m} + b$

Then the vertex weights form an arithmetic progression with difference two, namely $\frac{5m+5}{2}, \frac{5m+9}{2}, \dots \dots$

IV. CONCLUSION

This paper established the main double inequalities on chromatic number corresponding to magic and anti-magic graphs. In addition, the inequalities on chromatic number associated with magic and anti-magic graphs are satisfied.

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