A Characteristic Queue Model to an Interdependent Communication under Fuzzy Environment

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Abstract—
This paper deals an interdependence queue model to a communication system under fuzzy environment. Here the data and voice are integrated with statistical multiplexing. On the condition of overload, the transmission of packet is reduced slowly in the multiplex queue to relieve from congestion and transmitted over a secondary transmitter. The paper generalizes a system to construct the membership function of bulk arrival queues when the arrival rate and service rate are fuzzy numbers. The multiplexed message of data and voice is divided into packets of varying length packets and transmitted using trapezoidal fuzzy parameters based on Zadeh's extension principle. The model has reduced the average delay of transmission process and variability of the buffer content with uncertainty and finds its application in real-time environment.

***Keywords*— Steady state, Trapezoidal fuzzy number, Fuzzy set theory, average delay, Packet switching.**

I. Introduction

For any graph H=(U,D), the Lict graph η(H) having the vertex set as the union of the set of edges and the cut vertices set of H in which two vertices of η(H) are said to be adjacent when the associated edges or members of H are adjacent or incident. The distance d(x,y) between two vertices x and y in a connected graph H is the length of a shortest x-y path in H [2], [6]. It is well known that this distance is a metric on the vertex set U(H). For a vertex v of H, the eccentricity d(v) is the distance between v and a vertex farthest from v. The minimum eccentricity among the vertices of H is the radius, rad H, and the maximum eccentricity is its diameter, diam H. A x-y path of length d (x,y) is called a x-y geodesic [4]. Let F(x,y) is defined to be the set Queue Models can be studied mathematically since Queue Models afford the basic construction of a communication system due to the existence of noted contiguous similarity between the queue networks and the communication system. The input arriving messages, are assumed as customers/ jobs, intermediate buffers as waiting contours and all the related operations in transmission of the message can be considered as services, buffer capacity is viewed as the queue system's capacity while the type of service is first in first out (FIFO). From the user's view, the data is transmitted from user memory on the sending node to user memory on the receiving node. During transmission the communication library make a copy of transmission temporarily in system memory. This process is called buffering. Data may undergo buffering either at the sending node/ receiving node or both. In the situations of differences in the receiving and processing rate or both the receiving and processing rates are varying with respect to time (online video stream), the buffers are typically used. Buffer adjusts timing often by employing Queue algorithm in memory with the simultaneous operation of writing data and reading data both at different rates. Queue Models provide more accurate analysis of modern communication system. These models play vital role in modeling of voice calls. With the arrival of faxes and internet, the characteristic of traffic has changed desperately. This reason made packet switched networks to come into existence and have acquired importance over circuit switching networks. The Circuit switching telephone is found to be not suitable to handle interactive traffic due to its capability for less common service requests with lengthy holding times. On the other hand, packet switched networks permits any host to transmit data to any other host with no prior reservation of the circuit. Multiple paths exist between the two hosts and all the packets associated to a transmission may/ may not take the same route in a packet switched network. Whenever the source has data to send, one path is selected on-demand basis between sender and receiver and sender converts the data into packets and forwards to next computer/ router. The router temporarily stores this packet until the receiver/ output line is free and then, transfer the packets to next computer/ router (hop). Thus data move hop by hop until it reaches the intended destination. Thus packet switching is the more efficient network in terms of network bandwidth. In circuit switching network, a dedicated circuit cannot be used by others, till the source and destination leave the circuit. This leads to wastage in network bandwidth. In communication system using packet node, the network resources are handled by statistical multiplexing/ dynamic memory allocation where the communication channel is efficiently divided into an inconsistent number of data streams. Also, statistical multiplexing reduces the delay in packet switching in a communication system. Recently, researchers have been committed in the field of communication system, data traffic process in the framework of queuing analysis. In general, it is pretended that the processes of arrival and service are independent but there are numerous real-time situations in which there exist an association between them. Most of the research works in this area analyzed the communication network as interconnected queues with an assumption that arrival and service designs are interdependent [1]. A stochastic queue model is developed to interdependent communication network in which arrival messages are geometrically distributed and transmission at the node are correlated adopting a bi-variate Poisson process [3, 4] and [5]. The average transmission delay and variance of packets in buffer are calculated using Chapman Kolmogorov equation [5]. The interdependent queuing model is explored to a communication system with voice packets and data under fuzzy environment [6]. Fuzzification of the bulk queuing model of a communication system with statistical multiplexers containing the packet voice source and the data is performed [6]. Few research works attempted to develop and analyze an interdependent communication networks (teleprocessing, packet radio systems, store and forward data/ voice transmission,etc.) on performance of the communication system. Hence it is necessary to take into account these dependent characteristics of the processes of arrival and service for any practical analysis.

II.MODEL DESCRIPTION & NOTATION

The arrival of packets and number of transmission are assumed to be correlated. Both the arrival of packets and number of transmission follows a bivariate poisson process with joint probability mass function Milne [1974] & Rao Srinivasa, et al. [2000]. It is also assumed that the capacity of buffer is infinite and the number of arrival of packets is given by a random variable σ in fuzzy environment.

2.1 Notation

$\ddot{I}_\sigma$: Arrival rate of message having σ packets in fuzzy.

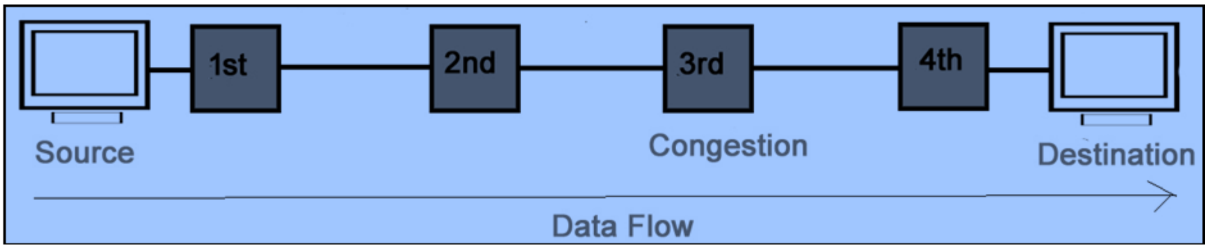
$\dot{C}_\sigma$: Covariance between arrival of packets and number of transmission completed in fuzzy.

Ă: Average transmission rate in fuzzy.

$\check{r}_\sigma$: Probability that a batch size of σ packets will arrive to buffer in fuzzy environment.

The composite arrival rate of packets $\ddot{I} = \sum_\sigma \ddot{I}_\sigma$ and the covariance of the composite arrivals and transmission completions $\dot{C} = \sum_\sigma \dot{C}_\sigma$.

The covariance $\acute{C}$ is obtained by means of bit dropping method (flow control) inducing the dependence between message arrival and message transmissions. In a communication system, the process of flow control is to manage the transmission rate between sender and receiver in order to prevent a fast sender from overpowering a slow receiver. Congestion control is used for controlling the data flow when congestion has absolutely occurred.



III. MATHEMATICAL ANALYSIS

The model differential difference equation of the fuzzy system in transient form can be given in equations 1 and 2 as:

$$P_\sigma{}'(t) = -\breve{I} + \ddot{A} - 2\acute{C}P_m(t) + \ddot{A} - \acute{C}P_{m+1}(t) + \breve{I} - \acute{C}\sum_{n=1}^{m} P_{m-n}(t)\check{I}_n \quad m \ge 1$$

(1)

$$P_0{}'(t) = -\breve{I} - \acute{C}P_0(t) + \breve{A} - \acute{C}P_1(t)$$

(2)

The steady state condition is attained when the nature of the system is independent of time. The steady state equations are $t \to \infty$.

$$0 = -\breve{I} + \breve{A} - 2\acute{C}P_m(t) + \breve{A} - \acute{C}P_{m+1}(t) + \breve{I} - \acute{C}\sum_{n=1}^{m} P_{m-n}(t)\check{I}_n \quad m \ge 1$$

(3)

$$0 = -\breve{I} - \acute{C}P_0(t) + \breve{A} - \acute{C}P_1(t)$$

(4)

Assuming Probability generating function of the number of packets in the buffer and the number of packets that the message is divided:

$$\acute{P}(\acute{z}) = \sum_{m=0}^{\infty} \acute{P}_m \acute{z}^m, |\acute{z}| \le 1$$

$$\check{I}(\acute{z}) = \sum_{m=0}^{\infty} \check{I}_m \acute{z}^m, |\acute{z}| \le 1$$

Multiplying equation 2 with $\acute{z}^m$ and letting $n = 0 \ to\ \infty$. Simplifying the expression,

$$0 = -(\breve{I} - \acute{C})\acute{P}(\acute{z}) - (\ddot{A} - \acute{C})[\acute{P}(\acute{z}) - \acute{P}(0)] + \frac{(\ddot{A} - \acute{C})}{2}[\acute{P}(\acute{z}) - \acute{P}(0)] + (\breve{I} - \acute{C})\check{I}(\acute{z})\acute{P}(\acute{z})$$

(5)

From equation 5,

$$\acute{P}(\acute{z}) = \frac{(\ddot{A} - \acute{C})(1 - \acute{z})}{(\ddot{A} - \acute{C})(1 - \acute{z}) - (M - \acute{C})\acute{z}(1 - \check{I}(\acute{z}))} P_0{}'$$

(6)

Assuming $\check{I}_\sigma$, the batch size is geometrically distributed, i.e., $\check{I}_\sigma = (1 - \beta)\beta^n \quad with \quad 0 < \beta < 1$, then

$$\check{I}(\acute{z}) = \frac{(1 - \beta)\acute{z}}{1 - \beta\acute{z}}$$

(7)

Substituting equation 7 in 6,

$$P(z) = \frac{(\ddot{A} - \acute{C})(1 - \beta\acute{z})P_0{}'}{(\ddot{A} - \acute{C})(1 - \beta\acute{z}) - (M - \acute{C})\acute{z}}$$

(8)

For $(\breve{I} - \acute{C}) < (\ddot{A} - \acute{C})(1 - \beta)$

(9)

IV. PERFORMANCE MEASURE

Using condition $\acute{P}(1) = 1$, Probability that the system is empty i.e., $\acute{P}(0) = 1 - \frac{(\breve{I}-\acute{C})}{(\breve{A}-\acute{C})(1-\beta)}$ which is given by

$\acute{P}(0) = 1 - \hat{\delta}_0$ (10)

Expanding $\acute{P}(\acute{z})$ and collecting the coefficient of $\acute{z}^{\lambda}$, the probability that the size of the system is λ as

$\acute{P}_{\sigma} = \left(1 - \hat{\delta}_0\right)[\beta + (1-\beta)\hat{\delta}_0]^{\lambda-1}(1-\beta)\hat{\delta}_0$ (11)

If $\beta \to 0$, then

$\acute{P}_{\lambda} = (1-\beta)\hat{\delta}_0 \quad \lambda > 0$ (12)

This expression depicts the interdependent communication network with no bulk arrivals in fuzzy environment. The average number of packets in the fuzzy system is

(1) $N = \frac{\hat{\delta}_0}{(1-\beta)(1-\hat{\delta}_0)}$ where $\hat{\delta}_0 = \frac{(\breve{I}-\acute{C})}{(1-\beta)(\breve{A}-\acute{C})}$

(2) The variance of number of packets in the fuzzy system is

$Variance = \frac{\beta\hat{\delta}_0(1-\hat{\delta}_0)+\hat{\delta}_0}{(1-\beta)^2(1-\hat{\delta}_0)^2}$

The values of average number of packets (N) for $\beta = 0.1, 0.15, 0.2, 0.25$ and $\acute{C} = 0.1, 0.2, 0.25, 0.3$ for the fixed $\breve{I} = 3,4$ and $\breve{A} = 9,8$ respectively is shown in Fig.1

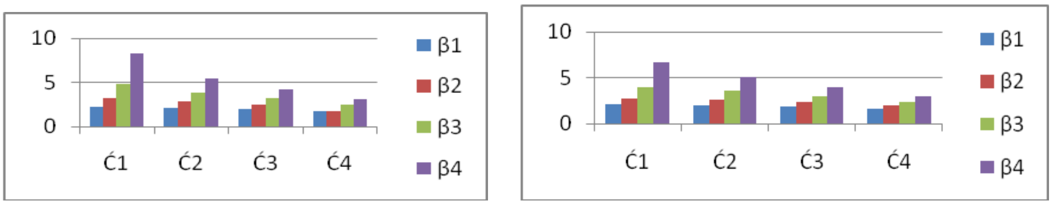


Fig.1 The values of average number of packets in the system

After defuzzified the average number of packets for $\beta = 0.1, 0.15, 0.2, 0.25$ and $\acute{C} = 0.1, 0.2, 0.25, 0.3$ for the fixed $\breve{I} = 3,4$ and $\breve{A} = 9,8$ respectively is shown in Fig.2.

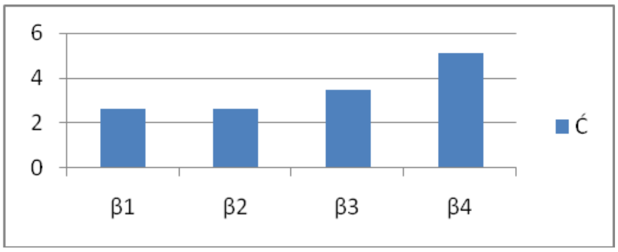


Fig.2 The values of average number of packets in the system after defuzzification

The variance of number of packets in the system for $\beta = 0.1, 0.15, 0.2, 0.25$ and $\acute{C} = 0.1, 0.2, 0.25, 0.3$ for the fixed $\breve{I} = 3,4$ and $\breve{A} = 9,8$ respectively is shown in Fig.3

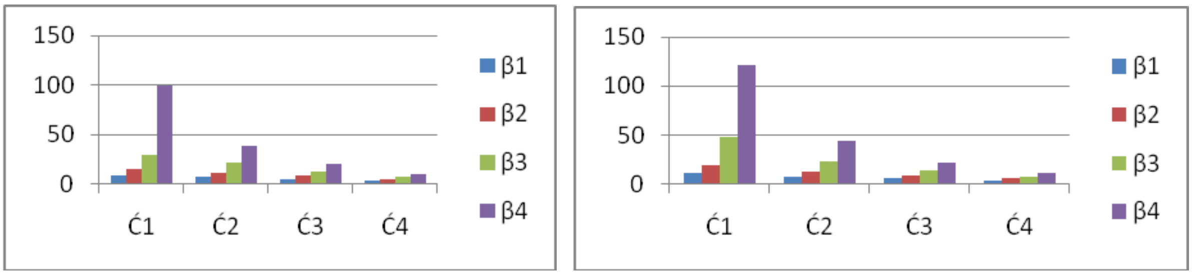


Fig.3 The variance of number of packets in the system

After defuzzification the Variance of number of packets in the system for $\beta = 0.1, 0.15, 0.2, 0.25$ and $\acute{C} = 0.1, 0.2, 0.25, 0.3$ for the fixed $\bar{I} = 3,4$ and $\breve{A} = 9,8$ respectively is shown in Fig.4.

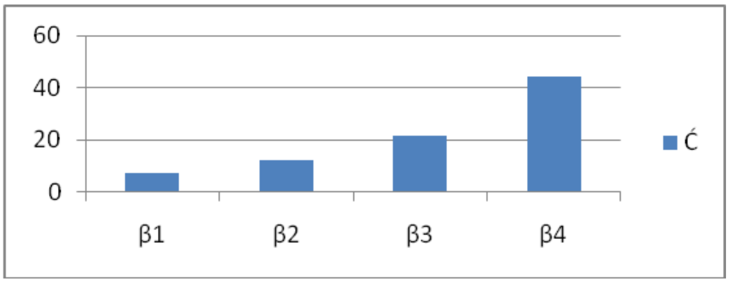


Fig.4 The variance of number of packets in the system after defuzzification

V.CONCLUSION

From figures 1 and 2, it is inferred that the average numbers of packets in the system and also in the buffer are reducing by increasing the independent parameters (positive) with the other parameters as fixed. In addition, it is found that the average number of packets in fuzzy system and in buffer is increasing as β increases for given value of $\bar{I}, \breve{A}$, and $\acute{C}$. From figures 3 and 4, the monotonic nature of the variability of number of packets in buffer is again observed. In general, as β increases, the variance also increases with the other parameters as fixed. Also on defuzzification , the result shows the similar behavior.

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