Number of Lict Graph and Bounds on Geodetic Number on Connection with Elements of Lict Graph

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Abstract :

The lict graph of H is represented by ρ(H) for any graph H=(U,D). The Lict graph ρ(H) of H is the graph with vertex set to be the union of edges and cut vertices of H where in which 2 vertices are said to be adjacent if and only if the associated edges of H are adjacent or the associated members are incident. For two vertices x and y of H, the set F(x,y) includes all vertices lying on a x-y geodesic in H. If T is a set of vertices of F, then F(T) is the union of all sets F(x,y) for vertices x and y in T. The geodetic number h(H) is the minimum cardinality among the subsets T of U(H) with F(T)=U(H). In this paper the geodetic number of lict graph of any graph is obtained. In addition, many bounds on geodetic number in connection with elements of H is obtained.

Keywords: Cross product, Distance, Edge covering number, Edge independent number, Geodetic number, Lict graph.

I. Introduction

For any graph H=(U,D), the Lict graph η(H) having the vertex set as the union of the set of edges and the cut vertices set of H in which two vertices of η(H) are said to be adjacent when the associated edges or members of H are adjacent or incident. The distance d(x,y) between two vertices x and y in a connected graph H is the length of a shortest x-y path in H [2], [8]. It is well known that this distance is a metric on the vertex set U(H). For a vertex v of H, the eccentricity d(v) is the distance between v and a vertex farthest from v. The minimum eccentricity among the vertices of H is the radius, rad H, and the maximum eccentricity is its diameter, diam H. A x-y path of length d (x,y) is called a x-y geodesic [4]). Let F(x,y) is defined to be the set of all vertices lying on some u-v geodesic of H, and for a nonempty subset T of U(H),

$$F(T) = \cup_{x,y\in T} F(x,y) \qquad (1)$$

A set T of vertices of H is called a geodetic set in H if F(T)=U(H), and a geodetic set of minimum cardinality is a minimum geodetic set (Chartrand et al [2001]). The geodetic number h(H) represents the cardinality of a minimum geodetic set in a graph (H). Now the geodetic number of lict graph of a graph (H) is defined [1]. A vertices set T′ of η (H)=G is represented as a geodetic set in G when F(T′)=U(G), and a geodetic set of minimum cardinality is a lict geodetic number of graph (H) denoted by $h_\eta(H)$. The Cartesian product of two graphs H, G by H × G , and it is the graph with the vertex set $V(H) \times V(G)$ specified by putting (x, y) adjacent to (x′ , y′) if and only if

(1) x= x′ and $yy' \in D(G)$ or

(2) y= y′ and $xx' \in D(H)$.

A vertex v is said to be an extreme vertex in a graph H, if the sub graph induced by its neighbours is complete. A set of vertices set covers all the edges of graph (H) is said to be the vertex cover in a graph G. The minimum number of vertices in a vertex cover of H is the vertex covering number $\alpha_0(H)$ of H. An edge cover of a graph H without isolated vertices is a set of edges of G that covers all the vertices of H [9]).

The edge covering number $\alpha_1(H)$ of 'G' is shown to be the minimum cardinality of $\alpha_1(H)$ of H. A set of vertices in H is independent if no two of them are adjacent. The largest number of vertices in such a set is called the vertex independence number of H and is denoted by $\gamma_0(H)$. An independent set of edges of H has no two of its edges are adjacent and the maximum cardinality of such a set is the edge independence number $\gamma_1(H)$.

II.PRELIMINARIES

The following theorems are needed for further results

Theorem 2.1. For any graph H of order m, $\sigma_1(H)+\gamma_1(H)=m$ [3].

Theorem 2.2. For a complete bipartite graph $L_{q,t}$, the edge covering number is $\sigma_1(L_{q,t})=t$ if $1\leq t\leq s$ (Chartrand [2001]).

Theorem 2.3. For any path Q_m, the edge covering number [7] is

$$h_\eta(Q_m)=\begin{cases} 2\sigma_1 Qm) = \frac{m}{2}, & m \text{ is even} \\ 2\sigma_1 Qm)\frac{m+1}{2}, & mi\ s\ odd \end{cases}$$

Theorem 2.4.Every geodetic set of a graph has its extreme vertices [6].

Theorem 2.5.If H is a nontrivial connected graph, then $h(H) \leq h(H \times L_2)$ [6].

The following propositions are explained initially.

Proposition 1.The pendent edges and cut vertices of a tree are the extreme vertices of a lict graph $\eta(H)$

Proposition 2. Geodetic number of $L_{k1}, L_{k2} \ldots L_{kn}$ 1 2 is the number of non cut vertices of H.

Proposition 3. For any path Q_m, $\eta(Q_m) = L_3 \cdot L_3 \ldots (m-2)$ factors.

III. MAIN RESULTS

Theorem 3.1. For any tree S with m vertices, $h_\eta(S) = m$.

Proof.

Let the set of entire extreme vertices be T of a lict graph of a tree S. By the Theorem 2.5, $h_\eta(S)\geq T$. But, for a vertex (internal) v of S, there exists end vertices u,v of S so that v exists on the unique u-v geodesic in S. Thus $v\in F(T)$ and F(T)= U(S), then $h_\eta(S)\leq T$. Therefore, $h_\eta(S)=T$. Also, every geodetic set T′ of S has T as the unique minimum geodetic set. From the section 2.5 and proposition 1, the pendent edges and cut vertices of S are the extreme vertices of H. Since, the number of vertices in S is the addition of cut vertices and the pendent edges and therefore the minimum geodetic set represents the number of vertices m in S. Thus $h_\eta(S)$ =m.

Corollary3.1.1. For any path Q_m with m vertices, the lict geodetic number $h_\eta(Q_m) = m$.

Proof. Proof is pursued from Theorem 3.1.

Theorem 3.2. For any S (tree), the lict geodetic number is $h_\eta(S) = \sigma_1(S) + \gamma_1(S)$.

Proof. From the section 2.1, $\sigma_1(S)+\gamma_1(S) = m$. Since $h_\eta(S) = m$, $h_\eta(S) = \sigma_1(S) + \gamma 1(S)$.

Theorem 3.3. For any star $L_{1,q}$, the geodetic number $h_\eta(L_{1,q}) = \sigma_1(L1,q) + 1$.

Proof. Let $d_1,d_2,d_3,\ldots,d_j$ be the pendant edges and b_1 be the cut vertex of $L_{1,q}$. Let S be the set of entire extreme vertices of H of $L_{1,q}$. Therefore $h_\eta(L_{1,q}) = T$. From the lict graph (H), $\eta(L_{1,j}) = L_{j+1}$. Also by the proposition 1, the pendant edges and the cut vertices of a $L1,q$ are the extreme vertices of H. Since $\sigma_1(L_{1,s}) = t$, then $h_\eta(L_{1,q}) = \sigma_1(L_{1,q})+1$.

Theorem 3.4. For any path Q_m with m vertices, the lict geodetic number $h_\eta(Q_m) = K(Q_m) + b_i$, where $K(Q_m)$ represents the line graph of Q_m and b_i represents the number of cutvertices.

Proof. Let Q_m be the path with $m \geq 3$ vertices & U and d be the vertices and edge set of Q_m respectively. But $K(Q_m) = Q_{m-1}$ & $h(Q_m) = 2$ from the theory of line graph. Since $K(Q_m)$ is a sub graph of h_η Q_m , from the lict graph, cut vertices b_i are adjacent to the vertices $d_1,d_2,d_3,\ldots,d_j$ of $K(Q_m)$. Hence $h_\eta(Q_m) = KQ_m + b_i$.

Theorem 3.5. For any path Q_m with n vertices, the geodetic number

$$h_\eta(Q_m) = \begin{cases} 2\sigma_1 Qm), & m \text{ is even} \\ 2\sigma_1 Qm), & m \text{ i s odd} \end{cases}$$

where $\sigma_1(Q_m)$ is an edge covering number.

Proof. Let Q_m , U and d takes the same definition as in previous section From the section 2.3, the number of edge covering $\sigma_1(H)$ of a graph is a minimum cardinality of an edge cover of H.

Case (1). For even m, and from 2.3, $\sigma 1(Q_m) = m/2$

$$= m = 2\,\sigma 1(Q_m)$$

Since $h_\eta(Q_m) = m$, $h_\eta(Q_m) = 2\,\sigma 1(Q_m)$

Case (2). For odd m, and from 2.3, $\sigma 1(Q_m) = (m+1)/2$

$=m+1=2\,\sigma 1(Q_m)$

Since $h_\eta(Q_m) = m$, $h_\eta(Q_m) = 2\,\sigma 1(Q_m) - 1$.Hence the result.

Theorem 3.6. For any path Q_m with m vertices, the lict geodetic number of $L_2 \times H$ is $h_\eta(L_2 \times \eta\,(Q_m)) = m$.

Proof. From the lict graph, $\eta(Q_m) = L_3 \cdot L_3 ...(m-2)$ factors for $m \geq 3$. Consider $H= \eta(Q_m)$. Let the graph $L_2 \times H$ is obtained from H_1 and H_2 of H and T represents a minimum geodetic set of that graph. Let $v \in U$ exists on some geodesic. Since T is a geodetic set, and from 3.1 at least one of u and v become the part of U_1. If both $u,v \in U_1$ then $u, v \in T'$. Hence, it is essential to assume that $u \in U_1$, $v \in U_2$. If y associates to u then $y=u \in T'$. Hence, assume y corresponds to $v' \in T'$, where $v' \neq u$. Since $d(u,v)=d(u,v')+1$ and the vertex v lies on an u-v geodesic in $L_2 \times H$. Therefore v lies on an u-v geodesic in H i.e. $h(H) \leq h(L_2 \times H)$. Interchangeably, assume T contains a vertex x with the property that every vertex of H_1 lies on an u-z geodesic H_1 for some $z \in T$. Let T′ has 'u' together with vertices of H_2. Thus $|T'| = |T|$. Hence, it is shown that T′ is a geodetic set of $L_2 \times H$. Hence $h(L_2 \times H) \leq h(H)$. Thus $h_\eta\,(L_2 \times \eta(Q_m)) = m$.

Corollary3.6.1. For any path Q_m having 'm' vertices, the lict geodetic number '$h_\eta\,(L_3 \times \eta(Q_m)) = m$'.

Proof. Proof follows from Theorem 3.6.

Theorem 3.7.

$h_\eta\,(H') \geq 3$ such that H′ be the sum of a pendant edge to a cycle H.

Proof. Let d_1 be a cycle with 'm' vertices (odd) and assume H′ be the graph realized from $H= Q_m$ by the sum of a pendant edge {x y} such that $y \notin H$, $x \in H$. From the lict graph, η(H′) has L_4 as an induced sub graph, also the edge $(x,y)= d_1$ be a vertex of η (H′). In addition, the cut vertex 'u' is in turn a vertex of η(H′) and it is a portion of some geodetic set of η (H′). Let d_i equal to a set of o and p belongs to H such that e (x,o) in the graph η(H′), two elements subset of T of η(H′) defines that $F(T) = U[\eta\,(H')]$. Thus $h_\eta\,(H') \geq 3$.

Theorem 3.8. $h_\eta\,(H') = h(H') + h(H) = 4$ such that H′ be the sum of pendant edge to a cycle H.

Proof. Let d_1 be a cycle with 'm' vertices (even) and let H′ be the graph obtained from $H=B_m$ by the sum of a pendant edge {x, y} such that $y \notin H$, $x \in H$. From the lict graph, η (H′) has L_4 as an induced sub graph, also the edge $(x, y)=d_1$ is a vertex of η (H′). Additionally, the cut vertex 'u' is a vertex of η (H′). Hence the vertices of η(H′) are a part of it. Thus, $\eta(H')=Bn \cup L4$ & $h(H') = 2 = h(H)$. Hence $h_\eta\,(H') = h(H') + \eta(H) = 4$.

Theorem 3.9. Let H′ be the graph realized by the sum of a pendant edge $x_i\, y_j$, i=1,2,3,4…m, j=2,3,….l to each vertex of H such that $y\, j \notin H$, $x_i \in H$ then $h_\eta\,(H') = h(H') + l$ where l be the number of pendant vertices of H.

Proof. Let d_1 be a cycle and $H=B_m$ with 'm' vertices and graph H′ be realized by the sum of a $\{x_i,y_j\}$, i=1,2,3,4…m, and j=2,3,….l to every vertex of H so that $y_j \notin$ H and $xi \in$ H. Assume l and x_i be the pendant vertices & cut vertices in H′ . From the lict graph, η (H′) have 'l' copies of L_4 as an induced sub graph. If $(x_i, y_j) = b_i$ then $d_1,d_2,d_3,.....d_m$ and x_i becomes the vertices of η(H′) and these elements are a portion of some geodetic set of η(H′) say h_η (H′).

Now let T′ is the geo-dominating set of H′. Therefore 'x_i', 'y_j' and 'd_{ij}' are the extreme vertices of η(H′) . From Theorem 3.1, $h_\eta (H') = h(H') + l$.

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