AN ECONOMIC ORDER QUANTITY MODEL FOR DETERIORATING ITEM WITH NON-LINEAR DEMAND UNDER INFLATION, TIME DISCOUNTING AND A TRADE CREDIT POLICY

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Abstract: *This paper discusses an inventory model based on the discounted cash flows(DCF) approach for the analysis of the optimal ordering policies in presence of inflation and trade-credit policy of the type* α/M_1 *net* M*. The effects of deterioration and time-value of money are also taken into account. The demand rate is assumed to be non-linear over time. The results are illustrated with two numerical examples. Optimal present worth of all future cash flows in discount case is economically beneficial. Sensitivity analysis of the optimal solution with respect to changes in the parameters of the system is carried out.*

Keywords: Deterioration, Non-linear demand, Inflation, Trade-Credit Policy.

1. Introduction

Most of the classical inventory models did not take into account the effect of inflation and time value of money. Perhaps it was believed that inflation would not influence the cost and price components to any significant degree. But during the last 37 years, the economic situation of most of the countries has changed to large-scale inflation and consequent sharp decline in the purchasing power of money. Buzacott (1975) and Misra (1975) were the first to develop EOQ models taking inflation into account. Both of them considered a constant inflation rate for all the associated costs and minimized the average annual cost to derive an expression for the economic lot

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size. Their work was extended by researchers like Chandra and Bahner (1985), Aggarwal (1981), Misra (1979), Bierman and Thomas (1977), Sarker and Pan (1994), Bose *et al.* (1995), Ray and Chaudhuri (1997), Mirzazadeh *et al.* (2009), Hou and Lin (2006), Hou (2006), Chen (1998), Chung and Lin (2001), Horowitz (2000), Chen and Chen (2008), Xu (2008), Bera *et al.* (2009), Sharma (2009), Goswami *et al.* (2010), Tayal *et al.* (2015), Sharma *et al.* (2015), Mandal *et al.* (2015), Tyagi *et al.* (2014), Bera *et al.* (2013), Palanivel and Uthayakumar (2015), Kumar and Goswami (2015), etc., to cover considerations of time-value of money, different inflation rates for the internal and external costs, finite replenishment rate, shortage, etc.. Assumptions of different inflation and discount rates are justified in the particular case of Ray and Chaudhuri's(1997) model. So, researchers can not ignore the impact of inflation and time value of money.

As a standard practice in the real markets, a supplier usually allows a certain fixed period (known as the credit period) for settling the amount of money that the retailer owes to him for the items supplied. This TCP (trade credit policy) plays an important role in the business of many products for many reasons and it serves the interests of both supplier and retailer. For a supplier offering TCP, this is an effective means of price discrimination which circumvents anti-trust measures and is also an efficient method to stimulate the demand of the product. The supplier usually expects the profit to increase since rising sales volume compensates the capital losses incurred during the credit period. Also, the supplier finds an effective means of reducing the cost of holding stocks. On the other hand, the retailer earns an interest by investing the sale proceeds earned by him during the credit period. The benefit of TCP is more pronounced when the demand of a product depends on its retail price. Under such a marketing situation, the retailer is able to choose the selling price from a wider range of options existing in the market by utilizing the credit period offered by the supplier. TCP is a common industrial policy usually followed by wholesale marketers in selling many products. For these reasons, inventory modellers felt the need to take TC policies into consideration.

In developing mathematical inventory model, it is assumed that payments will be made to the supplier for the goods immediately after receiving the consignment. In day-to-day dealings, it is observed that the suppliers offer different *trade credit* policies to the buyers. One such trade credit policy prevalent in the market is α/M_1 net M which implies that a α% discount on sale price is granted by the seller to the buyer if payments are made within M_1 days while the full sale price is due within $M(>M_1)$ days from the date of invoice if the discount is not taken.

Ben-Horim and Levy (1982), Chung (1989), Aggarwal and Jaggi (1994), Mahata *et al.* (2013) , etc., devoted their attention to it. Aggarwal *et al.* (1997) discussed an inventory model taking into consideration inflation, time-value of money and a trade credit policy of the type α/M_1 net M with a constant demand rate. One of the results of their studies is that the EOQ is independent of the length of the credit period. This result is unexpected, perhaps due to their assumption of constant demand.

In the present paper, an attempt has been made to extend the model of Aggarwal *et al.* (1997) to an inventory of items deteriorating at a constant rate θ ($0< \theta <1$). Also the demand rate is taken to be linearly time-dependent instead of a constant demand rate in Aggarwal *et al.* (1997)). Sensitivity of the optimal solution is examined to see how far the output of the model is affected by changes in the values of its input parameters.

2. Assumptions and notations

We adopt the following assumptions and notations for the models to be discussed:

(i) $D(t) = (a - b\rho^t)$ is the demand rate at any time $t\geq 0$, $a> b > 0$ and $0 < \rho < 1$.

(ii) T is the replenishment cycle length.

(iii) Q_i is the order quantity in the $(i+1)^{th}$ cycle .

(iv) Replenishment is instantaneous.

(v) Lead time is zero.

(vi) Inventory carrying charge is I per unit per unit time.

(vii) Shortages are not allowed.

(vii) $C(0)$ and $A(0)$ are respectively the unit cost of the item and ordering cost per order at time zero.

(viii) h is the inflation rate per unit time.

(ix) r is the opportunity cost per unit time.

(x) a constant θ $(0 < \theta < 1)$ of the on-hand inventory deteriorates per unit of time.

(xi) The time horizon of the inventory system is infinite,

(xii) The term of credits policy is α/M_1 net M, which means that a α percent discount of sale price is granted if payments are made within M_1 days and the full sale price is due within M days from date of invoice if the discount is not taken.

3 Mathematical formation

Let $A(t)$ and $C(t)$ be the ordering cost and unit cost of the item at any time t. Then $A(t) = A(0)e^{ht}$ and $C(t) = C(0)e^{ht}$, assuming continuous compounding of inflation.

Let $I_i(t)$ be the instantaneous inventory level at any time t in the $(i+1)^{th}$ cycle. The differential equation governing the instantaneous state of $I_i(t)$ in the interval $[iT,\ (i+1)T]$ is

$$\frac{dI_i(t)}{dt} + \theta I_i(t) = -(a - b\rho^t), \qquad iT \leq t \leq (i+1)T \tag{1}$$

with $I_i(iT) = Q_i$ and $I_i((i+1)T) = 0,\ \ i = 0,\ 1,\ 2,....$

The solution of equation (1) with boundary condition $I_i(iT) = Q_i$ (see Appendix I) is

$$\begin{aligned} I_i(t) &= -\frac{a}{\theta} + \frac{b}{\theta + log\rho}\rho^t + Q_i e^{\theta(iT-t)} \\ &\quad + \frac{a}{\theta}e^{\theta(iT-t)} - \frac{b}{\theta + log\rho}\rho^{iT}e^{\theta(iT-t)} \\ &= -A + B\rho^t + Q_i e^{\theta(iT-t)} + Ae^{\theta(iT-t)} \\ &\quad - B\rho^{iT}e^{\theta(iT-t)}, \quad iT \leq t \leq (i+1)T, \quad i = 0, 1, 2.... \end{aligned} \tag{2}$$

where

$$Q_i = A(e^{\theta T}-1)+B\rho^{iT}(1-\rho^{iT}e^{\theta T}), \quad i=0,1,2,... \tag{3}$$

$$(\text{taking } A=\tfrac{a}{\theta}, \quad B=\tfrac{b}{\theta+log\rho}, \quad \theta+log\rho\neq 0)$$

Let t_0, t_1, t_2,... be the replenishment points and $t_{i+1}-t_i=T$ so that $t_i=iT$, $i=$ 0, 1, 2,....

3.1 Case I: when discount is taken

Here purchases made at time t_i, are paid after M_1 days and purchase price in real terms for Q_i units at time t_i in the $(i+1)^{th}$ cycle is

$$=Q_i(1-\alpha)C(t_i)e^{-hM_1}$$

The present worth of cash-flows for the $(i+1)^{th}$ cycle is

$$\begin{aligned} PV_i^{(d)}(T) &= [A(t_i)+Q_iC(t_i)(1-\alpha)e^{-hM_1} \\ &\quad +IC(t_i)(1-\alpha)e^{-hM_1}\int_{iT}^{(i+1)T}I_i(t)e^{-rt}dt]e^{-rt_i} \\ &= A_1+A_2+A_3, \quad \text{say} \end{aligned}$$

where

$$\begin{aligned} A_1 &= A(t_i)e^{-rt_i} \\ &= A(0)e^{-iRT}, \quad (\text{taking } \ R=r-h \ \text{ and using } \ t_i=iT), \\ A_2 &= Q_iC(t_i)(1-\alpha)e^{-hM_1}e^{-rt_i} \\ &= AF(e^{\theta T}-1)e^{-iRT}+BF\rho^{iT}(1-\rho^Te^{\theta T})e^{-iRT}, \quad (\text{Taking } \ F=C(0)(1-\alpha)e^{-hM_1}), \\ A_3 &= [IC(t_i)(1-\alpha)e^{-hM_1}\int_{iT}^{(i+1)T}I_i(t)e^{-rt}dt]e^{-rt_i} \end{aligned}$$

Now,

$$\begin{aligned} \int_{iT}^{(i+1)T}I_i(t)e^{-rt}dt &= \frac{A}{r}e^{-irT}(e^{-rT}-1)+\frac{B}{log\rho-r}\rho^{iT}e^{-irT}(\rho^te^{-rT}-1) \\ &\quad -\frac{A}{\theta+r}e^{-irT}(e^{-rT}-e^{\theta T}) \\ &\quad +\frac{B}{\theta+r}\rho^{(i+1)T}(e^{-rT}-e^{\theta T}) \end{aligned} \tag{4}$$

Hence,

$$\begin{aligned} A_3 &= \frac{IFA}{r}(e^{-rT}-1)e^{-iPT} + \frac{IFB}{log\rho - r}(\rho^T e^{-rT}-1)\rho^{iT}e^{-iPT} \\ &-\frac{IFA}{\theta+r}(e^{-rT}-e^{\theta T})e^{-iPT} \\ &+\frac{IFB}{\theta+r}\rho^{(i+1)T}(e^{-rT}-e^{\theta T})e^{-iPT}, \quad i=0,1,2,... \text{ (taking } P=2r-h) \end{aligned} \quad (5)$$

Therefore,

$$\begin{aligned} PV_i^d(T) &= A(0)e^{-iRT} + AF(e^{\theta T}-1)e^{-iRT} \\ &+BF\rho^{iT}(1-\rho^T e^{\theta T})e^{-iRT} + \frac{IFA}{r}(e^{-rT}-1)e^{-iPT} \\ &+\frac{IFB}{log\rho - r}(\rho^T e^{-rT}-1)\rho^{iT}e^{-iPT} - \frac{IFA}{\theta+r}(e^{-rT}-e^{\theta T})e^{-iPT} \\ &+\frac{IFB}{\theta+r}\rho^T(e^{-rT}-e^{\theta T})\rho^{iT}e^{-iPT}, \quad i=0,1,2,... \end{aligned} \quad (6)$$

The present worth of all future flows $PV_\infty^d(T)$ is given by

$$\begin{aligned} PV_{(d)}^\infty(T) &= \sum_{i=0}^{\infty} PV_i^{(d)}(T) \\ &= \frac{A(0)+AF(e^{\theta T}-1)}{(1-e^{-RT})} + \frac{BF(1-\rho^T e^{\theta T})}{1-\rho^T e^{-RT}} \\ &+\frac{IFA}{1-e^{-PT}}[\frac{\theta e^{-rT}}{r(\theta+r)} + \frac{e^{\theta T}}{\theta+r} - \frac{1}{r}] \\ &+\frac{IFB}{1-\rho^T e^{-PT}}[\frac{\theta+log\rho}{(\theta+r)(log\rho-r)}\rho^T e^{-rT} \\ &-\frac{\rho^T e^{\theta T}}{\theta+r} - \frac{1}{log\rho - r}], \quad \text{(using Appendix II)} \end{aligned} \quad (7)$$

where $A=\frac{a}{\theta}$, $B=\frac{b}{log\rho+\theta}$ and $log\rho+\theta \neq 0$.

The solution of the equation $\frac{dPV_\infty^{(d)}(T)}{dT}=0$ gives the optimal value of T provided it satisfies the condition

$$\frac{d^2PV_\infty^{(d)}(T)}{dT^2} > 0. \quad (8)$$

Now, $\frac{dPV_\infty^{(d)}(T)}{dT} = 0$ yields the equation

$$\frac{[aC(0)(1-\alpha)e^{-hM_1}e^{\theta T}(1-e^{-RT}) - \{A(0) + \frac{a}{\theta}C(0)(1-\alpha)e^{-hM_1}(e^{\theta T}-1)\}Re^{-RT}]}{(1-e^{-RT})^2}$$
$$-C(0)(1-\alpha)e^{-hM_1}b\rho^T \frac{[e^{\theta T}(1-\rho^T e^{-RT}) - (1-\rho^T e^{\theta T})(R - log\rho)e^{-RT}]}{(1-\rho^T e^{-RT})^2}$$
$$+IC(0)(1-\alpha)e^{-hM_1}\frac{[\frac{1}{\theta+r}(e^{\theta T} - e^{-rT})(1-e^{-PT}) - \{\frac{1}{\theta+r}(\frac{e^{-rT}}{r} + \frac{e^{\theta T}}{\theta}) - \frac{1}{r\theta}\}Pe^{-PT}]}{(1-e^{-PT})^2}$$
$$+\frac{IC(0)(1-\alpha)e^{-hM_1}b}{(1-\rho^T e^{-PT})^2}[\frac{\rho^T}{\theta+r}(e^{-rT} - e^{\theta T})(1-\rho^T e^{-PT})$$
$$-\{\frac{\rho^T}{\theta+r}(\frac{e^{-rT}}{log\rho - r} - \frac{e^{\theta T}}{log\rho+\theta}) - \frac{1}{(log\rho+\theta)(log\rho - r)}\}(P - log\rho)\rho^T e^{-PT}] = 0 \quad (9)$$

This equation being highly non-linear con not be solved analytically. It can be solved numerically for given parameter values. Its solution gives the optimal value T^* of the replenishment cycle time T. Once T^* is obtained, we can get optimal order quantities Q_i^* $(i = 0, 1, 2,....)$ from (3) and $PV_\infty^{(d)}(T^*)$, the optimal present value of all future cash flows, from (7).

3.2 case II: when discount is not taken

Here purchases made at time t_i are paid after M days and purchase price in real terms for Q_i units at time t_i in the $(i+1)^{th}$ cycle is

$$= Q_i C(t_i)e^{-hM}$$

The present worth of cash-flows for the $(i+1)^{th}$ cycle is

$$\begin{aligned} PV_i^{(wd)}(T) &= [A(t_i) + Q_iC(t_i)e^{-hM} + IC(t_i)e^{-hM}\int_{iT}^{(i+1)T} I_i(t)e^{-rt}dt]e^{-rt_i} \\ &= A(0)e^{-iRT} + AC(0)e^{-hM}(e^{\theta T}-1)e^{-iRT} \\ &\quad +BC(0)e^{-hM}(1-\rho^T e^{\theta T})e^{-iRT} + \frac{IAC(0)e^{-hM}}{r}(e^{-rT}-1)e^{-iPT} \\ &\quad +\frac{IBC(0)e^{-hM}}{log\rho - r}\rho^{iT}e^{-iPT}(\rho^T e^{-rT}-1) \\ &\quad -\frac{IAC(0)e^{-hM}}{\theta+r}(e^{-rT}-e^{\theta T})e^{-iPT} \\ &\quad +\frac{IBC(0)e^{-hM}}{\theta+r}\rho^T(e^{-rT}-e^{\theta T})\rho^{iT}e^{-iPT}, \quad \text{i} = 0, 1, 2,.... \end{aligned} \quad (10)$$

where $A = \frac{a}{\theta}$, $B = \frac{b}{log\rho+\theta}$ and $log\rho + \theta \neq 0$

The present worth of all future cash-flows is

$$
\begin{aligned}
PV_{\infty}^{(wd)}(T) &= \sum_{i=0}^{\infty} PV_i^{(wd)}(T) \\
&= \frac{A(0) + AC(0)e^{-hM}(e^{\theta T} - 1)}{1 - e^{-RT}} + \frac{BC(0)(1 - \rho^T e^{\theta T})e^{-hM}}{1 - \rho^T e^{-RT}} \\
&\quad + IAC(0)e^{-hM}[\frac{\frac{\theta e^{-rT}}{r(\theta+r)} + \frac{e^{\theta T}}{\theta+r} - \frac{1}{r}}{1 - e^{-PT}}] \\
&\quad + IBC(0)e^{-hM}[\frac{(log\rho + \theta)\rho^T e^{-rT}}{(\theta + r)(log\rho - r)} - \frac{\rho^T e^{\theta T}}{\theta + r} \\
&\quad - \frac{1}{log\rho - r}]\frac{1}{1 - \rho^T e^{-PT}}, \quad \text{(using Appendix II)} \qquad (11)
\end{aligned}
$$

where $A = \frac{a}{\theta}$, $B = \frac{b}{\theta + log\rho}$ and $log\rho + \theta \neq 0$

The solution of the equation $\frac{dPV_{\infty}^{(wd)}(T)}{dT} = 0$ gives the optimal value of T provided is satisfies the condition

$$\frac{d^2 PV_{\infty}^{(wd)}(T)}{dt^2} > 0. \qquad (12)$$

Now, $\frac{dPV_{\infty}^{(wd)}(T)}{dT} = 0$ yield the equation

$$
\begin{aligned}
&[\frac{aC(0)e^{-hM}e^{\theta T}(1 - e^{-RT}) - \{A(0) + \frac{a}{\theta}C(0)e^{-hM}(e^{\theta T} - 1)\}Re^{-RT}}{(1 - e^{-RT})^2}] \\
&- \frac{bC(0)e^{-hM}\rho^T}{(1 - \rho^T e^{-RT})^2}[e^{\theta T}(1 - \rho^T e^{-RT}) \\
&- (1 - \rho^T e^{\theta T})(R - log\rho)e^{-RT}] \\
&+ \frac{IaC(0)e^{-hM}}{(1 - e^{-PT})^2}[\frac{1}{\theta + r}(e^{\theta T} - e^{-rT})(1 - e^{-PT}) \\
&- \{\frac{1}{\theta + r}(\frac{e^{-rT}}{r} + \frac{e^{\theta T}}{\theta}) - \frac{1}{r\theta}\}Pe^{-PT}] \\
&+ \frac{IbC(0)e^{-hM}}{(1 - \rho^T e^{-PT})^2}[\frac{\rho^T}{\theta + r}(e^{-rT} - e^{\theta T})(1 - \rho^T e^{-PT}) - \{\frac{\rho^T}{\theta + r}(\frac{e^{-rT}}{log\rho - r} \\
&- \frac{e^{\theta T}}{log\rho + \theta} - \frac{1}{(log\rho + \theta)(log\rho - r)})\}(P - log\rho)\rho^T e^{-PT}] = 0. \qquad (13)
\end{aligned}
$$

As commented before, this equation also needs to be solved numerically.

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4 Computational results

The present worths of all future cash flows are functions of single variable T but highly non-linear. Our objective is to determine T which minimise the present worths. We first solve equation (9) by bisection method to obtain T for given input parameters. Minimum present worth for Case I is calculated from expression (7). Using the same method , we solve equation (13) for T. The minimum present worth for Case II is determined from expression (11).

To illustrate, we consider the base example: $a = 50$, $b = 5$, $C(0) = 10$, $r = 0.04$, $I = 0.02$, $A(0) = 2000$, $M_1 = 30$, $h = 0.02$, $\alpha = 0.1$, $\theta = 0.01$ and $\rho = 0.5$ in appropriate units. The optimal solution for Case I is $T^* = 13.928009$ and corresponding optimal worth $PV_\infty^{(d)}(T^*) = 13802.535$. For $\alpha = 0$ and $M = 35$ in the base example, the optimal solution for Case II is $T^* = 13.896332$ and $PV_\infty^{(wd)}(T^*) = 13851.534$. In the computational results, it is found that the optimal present worth of all future cash flows $PV_\infty^{(d)}(T^*)$ in Case I is less than that of Case II.

5 Sensitivity analysis

Based on the numerical examples considered above, a sensitivity analysis of T^*, $PV_\infty^{(d)}(T^*)$, $PV_\infty^{(wd)}(T^*)$ is performed by changing (increasing of decreasing) one parameter at time by 25% and 50%, keeping the remaining parameters at their original values. In Table I, it is seen that the percentage change in $PV_\infty^{(d)}(T^*)$ is almost equal for both positive and negative changes of all the parameters except r and h. It is somewhat more sensitive for a negative change than an equal positive change of parameter r while it is more sensitive for a positive change than an equal negative change in h. Due to positive and negative percentage changes in a, $C(0)$, I, $A(0)$, h and θ, $PV_\infty^{(d)}(T^*)$ increases and decreases respectively. But this trend is reversed for the parameters b, r, M_1, ρ and α. From Table I it is clear that the model is highly sensitive to r, moderately sensitive to a, $C(0)$, $A(0)$, M_1, slightly sensitive to h, α, I, θ and insensitive to b, ρ. Behaviour of the parameters in Table II (for $\alpha = 0$) are nearly the same as in Table I.

Table I : Discount case

changing parameter	(%) change	T^*	$PV_\infty^{(d)}(T^*)$	(%) change in T^*	(%) change in $PV_\infty^{(d)}(T^*)$
a	+50	11.7033	18231.2112	-15.9731	+32.0860
	+25	12.6624	16050.5179	-9.0869	+16.2867
	-25	15.7177	11459.9778	+12.8495	-16.9719
	-50	18.5641	8969.8559	+33.2862	-35.0130
b	+50	13.9280	13783.9799	0	-0.1344
	+25	13.9280	13793.2572	0	-0.0672
	-25	13.9280	13811.8119	0	+0.0672
	-50	13.9279	13821.0892	-0.4×10^{-3}	+0.1344
$C(0)$	+50	11.7033	18212.6593	-15.9726	+31.9516
	+25	12.6624	16041.2411	-9.0869	+16.2195
	-25	15.7177	11469.2554	+12.8490	-16.9047
	-50	18.5641	8988.4114	+33.2862	-34.8785
r	+50	10.9795	9312.2751	-21.1700	-32.5321
	+25	12.2117	10974.3582	-12.3227	-20.4903
	-25	16.6015	19821.2108	+19.1953	+43.6056
	-50	22.0191	42658.2506	+58.0925	+209.0610
$A(0)$	+50	16.5043	15993.9777	+18.4972	+15.8771
	+25	15.3001	14931.7909	+9.8516	+8.1815
	-25	12.3154	12578.6742	-11.5782	-8.8669
	-50	10.3191	11206.4855	-25.9110	-18.8085
I	+50	13.3805	14053.4423	-3.9313	+1.8178
	+25	13.6443	13929.6841	-2.0373	+0.9212
	-25	14.2347	13671.6880	+2.2016	-0.9480
	-50	14.5677	13536.7890	+4.5930	-1.9253

Table I continue.....

changing parameter	(%) change	T^*	$PV_\infty^{(d)}(T^*)$	(%) change in T^*	(%) change in $PV_\infty^{(d)}(T^*)$
M_1	+50	15.7984	11381.2093	+13.4292	-17.5426
	+25	14.8383	12516.6701	+6.5356	-9.3161
	-25	13.0658	15260.5009	-6.1903	+10.5630
	-50	12.2501	16915.5758	-12.0471	+22.5541
h	+50	17.8122	13980.8349	+27.8874	+1.2918
	+25	15.6972	13781.5362	+12.7022	-0.1521
	-25	12.4250	13992.9912	-10.7916	+1.3799
	-50	11.1324	14324.9898	-20.0717	+3.7852
α	+50	14.2690	13293.4984	+2.4484	-3.6880
	+25	14.0952	13548.6135	+1.2007	-1.8397
	-25	13.7670	14055.3106	-1.1560	+1.8314
	-50	13.6118	14306.9876	-2.2704	+3.6548
θ	+50	13.2207	14118.7288	-5.0781	+2.2908
	+25	13.5631	13961.7279	-2.617	+1.1538
	-25	14.3176	13641.0355	+2.7971	-1.1701
	-50	14.7348	13477.1078	+5.7924	-2.3577
ρ	+50	13.9402	13746.8815	$+0.872\times10^{-1}$	-0.4032
	+25	13.9288	13783.9910	$+0.57\times10^{-2}$	-0.1416
	-25	13.9279	13813.7165	-0.4×10^{-3}	+0.0810
	-50	13.9279	13821.4515	-0.4×10^{-3}	+0.1371

Table II : Without discount case

changing parameter	(%) change	T^*	$PV_\infty^{(wd)}(T^*)$	(%) change in T^*	(%) change in $PV_\infty^{(wd)}(T^*)$
a	+50	11.6761	18300.4200	-16.0170	+32.1184
	+25	12.6332	16109.7133	-9.0896	+16.3027
	-25	15.6826	11498.5229	+12.8541	-16.9874
	-50	18.5239	8997.5424	+33.3006	-35.0430
b	+50	13.8964	13832.8794	$+0.4\times10^{-3}$	-0.1347
	+25	13.8964	13842.2066	$+0.4\times10^{-3}$	-0.0673
	-25	13.8963	13860.8610	0	+0.0673
	-50	13.8963	13870.1881	0	+0.1347
$C(0)$	+50	11.6761	18281.7685	-15.9770	+31.9837
	+25	12.6332	16100.3867	-9.0896	+16.2354
	-25	15.6826	11507.8504	+12.8514	+16.9200
	-50	18.5239	9016.1976	+33.3006	-34.9083
r	+50	10.9551	9342.2693	-21.1655	-32.5543
	+25	12.1843	11011.3497	-12.3200	-20.5045
	-25	16.5633	19887.0237	+19.1916	+43.5727
	-50	21.9675	42833.0855	+58.0817	+209.2299
$A(0)$	+50	16.4678	16046.5497	+18.5047	+15.8467
	+25	15.2658	14982.6682	+9.8552	+8.1661
	-25	12.2869	12625.5375	-11.5817	-8.8510
	-50	10.2948	11250.8105	-25.9174	-18.7757
I	+50	13.3501	14103.0191	-3.9301	+1.8156
	+25	13.6133	13978.9751	-2.0367	+0.9201
	-25	14.2022	13720.3892	+2.2014	-0.9468
	-50	14.5345	13585.1856	+4.5925	-1.9229

Table II continue.....

changing para- meter	(%) change	T^*	$PV_\infty^{(wd)}(T^*)$	(%) change in T^*	(%) change in $PV_\infty^{(wd)}(T^*)$
M	+50	16.0940	11069.4872	+15.2060	-20.0848
	+25	14.9611	12359.8782	+7.6622	-10.7689
	-25	12.8971	15578.7099	-7.1909	+12.4692
	-50	11.9604	17581.8324	-13.9315	+26.9306
h	+50	18.1493	13579.8930	+30.6047	-1.9611
	+25	15.8278	13605.2447	+13.8990	-1.7781
	-25	12.2652	14279.6957	-11.7381	+3.0911
	-50	10.8712	14871.3269	-21.7690	+7.3623
θ	+50	13.1910	14168.4193	-5.0756	+2.2877
	+25	13.5326	14011.0761	-2.6177	+1.1518
	-25	14.2849	13689.9785	+2.7961	-1.1685
	-50	14.7010	13525.3870	+5.7902	-2.3546
ρ	+50	13.9087	13795.5912	$+0.887\times10^{-1}$	-0.4039
	+25	13.8971	13832.8913	$+0.61\times10^{-2}$	-0.1346
	-25	13.8963	13862.7757	0	+0.0812
	-50	13.8963	13870.5523	0	+0.1373

6 Concluding remarks

The model developed here deals with the optimal replenishment policy of a deteriorating item in presence of inflation and a Trade Credit Policy (TCP). It is different from the existing models in that the demand rate is taken to be time-dependent in contrast to a constant demand rate in the order models. A trended demand changes steadily over time. Demand of a consumer goods changes steadily fluctuation in the population density. A time-dependent demand rate is certainly more realistic than a constant demand rate. We can made a comparative study between the results of the discount case and without discount case. In the computational results it is found that the optimal present worth of all future cash-flows $PV_\infty^{(d)}(T^*)$ in Case I is

0.3537% less than that of Case II. Hence the discount case is considered to be better economically.

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Appendix I

From equation (1), we have

$$I_i(t) = -\frac{a}{\theta} + \frac{b}{\theta + log\rho}\rho^t + c_i e^{-\theta t}, \quad c_i \text{ being a constant and } i = 0,\ 1,\ 2,....$$

From the condition that $I_i(iT) = Q_i$, we have

$$Q_i = -\frac{a}{\theta} + \frac{b}{\theta + log\rho}\rho^{iT} + c_i e^{-i\theta T}$$

$$\text{or,} \quad c_i = [Q_i + \frac{a}{\theta} - \frac{b}{\theta + log\rho}\rho^{iT}]e^{i\theta T}, \quad i = 0, 1, 2, ...$$

Therefore,

$$I_i(t) = -\frac{a}{\theta} + \frac{b}{\theta + log\rho}\rho^t + Q_i e^{\theta(iT-t)} + \frac{a}{\theta}e^{\theta(iT-t)} - \frac{b}{\theta + log\rho}\rho^{iT}e^{\theta(iT-t)}$$

$$= -A + B\rho^t + Q_i e^{\theta(iT-t)} + Ae^{\theta(iT-t)} - B\rho^{iT}e^{\theta(iT-t)},$$

where $A = \frac{a}{\theta}$, $B = \frac{b}{\theta + log\rho}$, $\theta + log\rho \neq 0$, $iT \leq t \leq (i+1)T$, $\quad i = 0,\ 1,\ 2,....$

Again from the condition that $I_i((i+i)T) = 0$, we have

$$Q_i = A(e^{\theta T} - 1) + B\rho^{iT}(1 - \rho^T e^{\theta T}), \quad i = 0, 1, 2, ...$$

Appendix II

Assuming $r>h$, we have

$$\sum_{i=0}^{\infty} e^{-iRT} = \frac{1}{1 - e^{-RT}} \quad , \quad \sum_{i=0}^{\infty} e^{-iPT} = \frac{1}{1 - e^{-PT}};$$

$$\sum_{i=0}^{\infty} \rho^{iT} e^{-iRT} = \frac{1}{(1 - \rho^T e^{-RT})} \quad , \quad \sum_{i=0}^{\infty} \rho^{iT} e^{-iPT} = \frac{1}{(1 - \rho^T e^{-PT})};$$

since $\lim_{n\to\infty}(ne^{-(n+1)RT}) = 0$ and $\lim_{n\to\infty}(ne^{-(n+1)PT}) = 0.$