MHD Flow Of a Casson Fluid Between Rectangular Plate

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Abstract

In this paper, unsteady magneto hydrodynamics flow of an electrically conducting casson fluid bounded by rectangular non conducting plate is considered. An external uniform magnetic field is applied perpendicular to the plate and the fluid motion is subjected to uniform injection and suction. The lubrication, in the gap is taken to be a viscous incompressible and electrically conducting fluid. The gap separated by a film thickness, which is made up of nominal smooth and rough part. The numerical results for various physical parameters are discussed in terms of pressure distribution, Momentum and energy equations. The effect of Hall term is also studied.

Keywords *MHD flow, Hall Effect, Casson fluid.*

Introduction

The phenomenon of magentohydrodynamics flow with heat transfer from different geometries bounded by porous medium has several engineering and geophysical application such as, underground energy transport, thermal insulation, drying of porous plate etc. In addition to the above studies, some researchers have also focused on diverse application in astrophysics and geophysics, etc.

The study of couette flow in a rectangular channel of an electrically conducting viscous field, under the action of a transverse magnetic field has immediate applications in many devices such as, magnetohydrodynamics (MHD), power generators, MHD pumps, acceletors and fluid droplets sprays channel. Flows of a Newtonian fluid with heat transfer have been studied with or without Hall currents by many researchers [1-5]. Casson fluid is a shear thinning liquid which has an infinite viscosity at a zero rate of shear. At an infinite rate of shear casson, constitute a nonlinear relationship between stress and rate of strain which is found to be accurately applicable to printing inks [6-8]. Many researchers studied the flow and heat transfer of a casson fluid in different geometries [9]. The influence of the Hall current on the velocity and temperature fields have studied [10]. Unsteady Hartmann flow of a conducting Newtonian fluid between two infinite non-conducting horizontal parallel and porous plates have also studied [11-13]. The upper plate moves with a uniform velocity and lower plate is stationary. The fluid is acted upon by a constant pressure gradient and uniform magnetic field which is perpendicular to the plates.

The Hall current is taken into consideration while induced magnetic field is neglected due to Reynolds number. The two plates are kept at constant temperature. The joule and viscous dissipations are taken into account in the energy equation. The energy and momentum equation also play important role. When the gap between two matting surface becomes smaller the effects of roughness become more important. In most of applications, the smooth bearing surface would not be valid for accrete prediction of performance. Thus, surface roughness has been studied with much interest in recent years because bearing surfaces are rough to some extent. The local Nusselt and Sherwood number decrease with Brownian and thermophoresis parameters. The prandtl and Lewis number also show effect on this flow.

Geometrical Interpretation

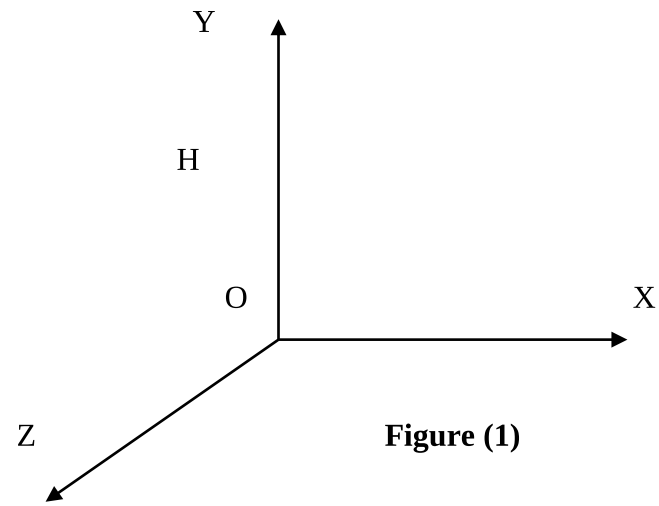

Figure (1)

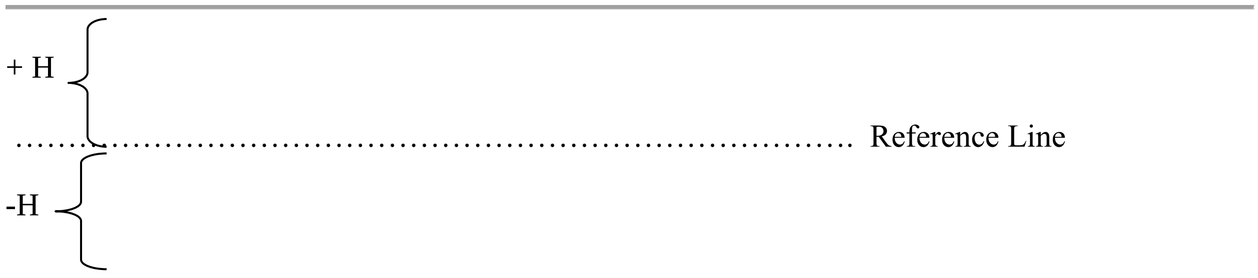

Figure (2)

We have selected a Cartesian coordinate system for the channel in such a way that x-axis is taken along the axial direction and y-axis is taken along transverse direction. The magnetic field H is perpendicular to the flow along y-axis.

Mathematical Formulation

The fluid is assumed to be laminar, incompressible and obeying a casson flow between two rectangular plate at the y = ± H (As shown in figure 1 and 2). The upper plate move with uniform velocity V_O and lower plate is stationary. The upper plate changes with temperature from t_1 to t_2 and t_2 to t_3 and so on. The fluid is acted upon by constant pressure. A uniform magnetic field H_o is applied in positive y direction and unaffected by magnetic Reynolds number, so the fluid velocity vector is given by

$$U\left(y,t\right)=u\left(y,t\right)i+vj+w\left(y,t\right)k \tag{1}$$

The MHD governing equation of motion in rectangular co - ordinates are

$$\frac{\partial p}{\partial x}=u_o\frac{\partial^2 u}{\partial y^2}-\eta\frac{\partial^4 u}{\partial y^4} \tag{2}$$

$$\frac{\partial p}{\partial y}=0 \tag{3}$$

$$\frac{\partial p}{\partial z}=v\frac{\partial^2 w}{\partial y^2}-\eta\frac{\partial^4 w}{\partial y^4} \tag{4}$$

When u, v, w are the velocities of fluid in *x, y, z* direction respectively. *P* is pressure distribution in the third region and υis viscosity of the fluid.

The fluid motion starts from rest at $t=0$ and no slip condition at the plates implies that the fluid velocity has neither a z nor an x component at $y=\pm H$ the initial temperature is assumed to be t_1, generally the flow of the fluid is governed by momentum equation.

$$\rho\frac{\partial v}{\partial t}=\nabla.(\upsilon\nabla V)-\nabla p+J\times H_o \tag{5}$$

Where ρ is density of the fluid

$$\upsilon=\left[K_e^2+\frac{T_o}{\sqrt{\frac{\partial u}{\partial y}+\frac{\partial w}{\partial y}}}\right]^2 \tag{6}$$

Where K_e^2 is the casson coefficient of viscosity and T_O is stress. If Hall term is present then current density "J" is given by.

$$J=\sigma\left\{u\times H_o-Y\left(J\times u_o\right)\right\} \tag{7}$$

Where σ^- is the electric conductivity of the fluid and Y is Hall factor so eq^n (6) can be written as

$$J\times H_o=\frac{\sigma H_o^2}{1+m^2}\left[\left(u+mw\right)i+\left(w-mu\right)k\right] \tag{8}$$

Where m is Hall parameter and given by $m = \sigma Y H_o$, there are two components of momentum Equation,

$$\rho \frac{\partial u}{\partial t} + \rho \upsilon_o \frac{\partial u}{\partial y} = -\frac{\partial p}{\partial x} + \frac{\partial}{\partial y}\frac{(\upsilon \partial u)}{\partial y} - \sigma \frac{H_o^2}{1+m^2}[u + mw] \quad (9)$$

$$\rho \frac{\partial w}{\partial t} + \rho \upsilon_o \frac{\partial w}{\partial y} = \frac{\partial}{\partial y}\frac{(\upsilon \partial w)}{\partial y} - \sigma \frac{H_o^2}{1+m^2}[w - mu] \quad (10)$$

The energy equation with viscous and joule dissipations is given by

$$\rho C_p \frac{\partial T}{\partial t} + \rho C_p \upsilon_o \frac{\partial T}{\partial y} = K \frac{\partial^2 T}{\partial y^2} + \mu\left[\left(\frac{\partial u}{\partial y}\right)^2 + \left(\frac{\partial w}{\partial y}\right)^2\right] + \sigma \frac{H_o^2}{1+m^2}\left[u^2 - w^2\right] \quad (11)$$

After introducing non dimensional quantities and applying boundary condition we get the equations in the form of

$$\frac{\partial u}{\partial t} + \frac{s}{R_e}\frac{\partial u}{\partial y} = -\frac{\partial p}{\partial x} + \frac{1}{R_e}\frac{\partial}{\partial y}\frac{(v\partial w)}{\partial y} - \sigma \frac{H_0^2}{1+m^2}[u + mw] \quad (12)$$

$$\frac{\partial w}{\partial t} + \frac{s}{R_e}\frac{\partial u}{\partial y} = \frac{1}{R_e}\frac{\partial}{\partial y}\frac{(v\partial w)}{\partial y} - \sigma \frac{H_0^2}{1+m^2}[u + mw] \quad (13)$$

$$\frac{\partial \theta}{\partial t} + \frac{s}{R_e}\frac{\partial \theta}{\partial y} = \frac{1}{P_r}\frac{\partial^2 \theta}{\partial y^2} + \frac{E_c v}{\partial y}\left[\left(\frac{\partial u}{\partial y}\right)^2 + \left(\frac{\partial w}{\partial y}\right)^2\right]\frac{H_a^2 E_c}{1+m^2}[u^2 + w^2] \quad (14)$$

$u = w = 0$ for $t \le 0$ and $u = w = 0$ at $y = -1, w = 0, u = 1$ at $y = 1$ for $t > 0$

$\theta = 0$ for $t \le 0$ and $\theta = 0$ at $y = -1, \theta = 1$ at $y = 1, \theta = 1$ at $y = 1$ for $t > 0$

$$\upsilon = \left[1 + \left(\frac{T_D}{\sqrt{\left(\frac{\partial u}{\partial y}\right)^2 + \left(\frac{\partial w}{\partial y}\right)^2}}\right)^{\frac{1}{2}}\right] \quad (15)$$

Results and discussion

We have calculated the value of $\frac{\partial u}{\partial t}$ and $\frac{\partial w}{\partial t}$ on the bases of following data

$\frac{\partial p}{\partial x} = 5$ (p varies from 1Pa to 5 Pa)

$p_r = 1$

$R_e = 1$

$H_a = 3$

$m = 3$ (m changes from1 to 3 numerical value)

s=1

As u and w change monotonically one by one substituting these values in equation (10)we get (-13), (-22), (-31), (-40) …………. similarly putting these values in equation (11) we get the value of $\frac{\partial w}{\partial t}$ = 4.5,9.1, 3.5,18…….

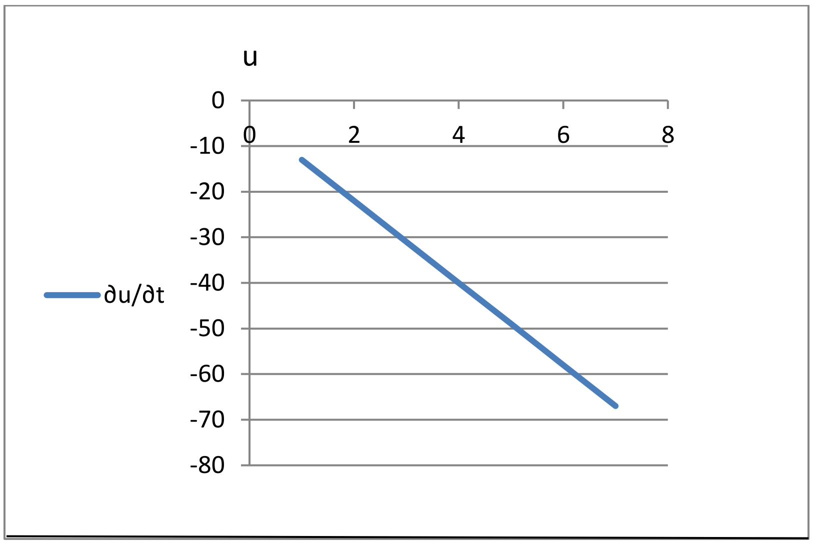

Figure: - (3) Graph is plotted u against $\frac{\partial u}{\partial t}$

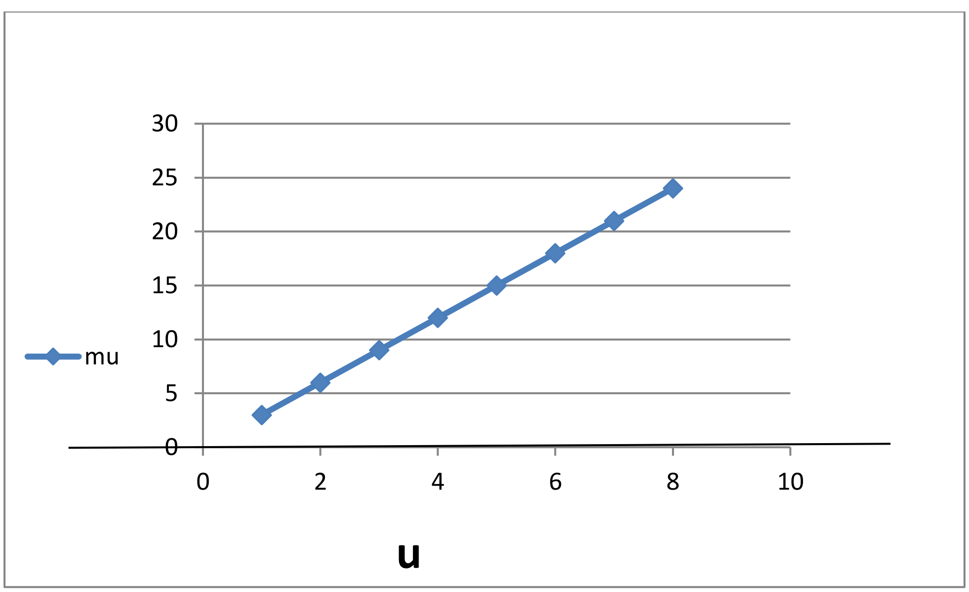

Figure: (4) Graph is plotted u against mu

Conclusion

The transient couette flow of a casson non - Newton fluid under the influence of on applied uniform magnetic field is studied considering the Hall Effect. The effects of casson number, Hall Parameter and suction parameter on velocities are studied. The Hall terms affect the main velocity component u in x direction and give rise to another velocity component w in z direction. It is observed that the velocity component u reaches the steady state faster than w. This is expected as u is source of w, while both u and w act as source of temperature. The reduction in u may in turn result in a small decrease in w. The source term of w is proportional to mu which decrease with increasing $\frac{mu}{1+m^2}$ (m>1). For small time, u and w are small, and an increase in m, increase u, but decrease in w. Then, the joule dissipation is also proportional to $\frac{1}{1+m^2}$ decreases. For large times, increasing m increases both *u* and *w*, in turn increases the joule and viscous dissipations.

Nomenclature

P_r Prandtl number
H_a Hartmann number
m Hall parameter
s Sherwood number
ν Viscosity
ρ Density of the fluids
σ Electrical conductivity
R_e Reynolds number
T_0 Stress
J Current density

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