Matrix method for deriving the response of a series Ł- C- R network connected to an excitation voltage source of constant potential

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Abstract:*The analysis of electric networks containing energy storage elements like a capacitor or an inductor or both a capacitor and an inductoris an essential course for most of the branches of the engineering. When we consider suchan electric network connected to an excitation source through a switch, thenon closing the switch and applying Kirchhoff's current* law or *Kirchhoff's voltage law, we get a differential equationwhose solution is generally obtained by adopting the classical method or Laplace transform. This paper presents a matrix method for getting the response of a series electric network of three passive elements namelyan inductor of inductance Ł, a capacitor of capacitance C, anda resistor of resistance R (i.e. a series Ł- C- R network), connected to an excitation voltage source of constant potential. The response obtained by solving the governing differential equation will provide an expression for the electric current which flows in theseries Ł- C- R network connected to an excitation voltage source of constant potential. The nature of this electric current is determined by the circuit elements-inductor Ł, capacitor C, andresistor R.*

Keywords**:***Electric network, Excitation source, electric current, Response.*

Introduction

Active electric elements are defined as those which have the ability to deliver average electric power greater than zero to the external electric devices in an infinite time interval whereas, Passive electric elements are defined as those which do not have the ability to do so. The electric circuit of a series Ł- C- R network consists of three passive electric elements namely an inductor Ł, a capacitor C, and a resistor R, connected in series to an active electric element namely a voltage source.It is used as atuning circuit, which is an example of band pass filtering, or resonant circuit in the radioand television sets to tune or resonate a particular frequency band from the wide range of radio frequency components.

Eigenvalues and Eigenvectors:

Let D be a matrix of order n x n with d_{ij}as its elements, then a column matrix T and a constant μcan be foundsuch that DT - μT = 0 or $|D - \mu I|T = 0$

This equation on simplifying will provide n homogeneous linear equations as written below:

$(d_{11} - \mu)\, t_1 + d_{12}\, t_2 + \cdots + d_{1n}\, t_n = 0$

$d_{21}\, t_1 + (d_{22} - \mu)\, t_2 + \cdots + d_{2n}\, t_n = 0$

$d_{31}\, t_1 + d_{32}\, t_2 + (d_{33} - \mu)\, t_3 \ldots + d_{3n}\, t_n = 0$

……………………………………………..

$d_{n1}\, t_1 + d_{n2}\, t_2 + \cdots + (d_{nn} - \mu)\, t_n = 0.$

These equationswill have a non – trivial solutionif the determinant of their coefficients is zero. In simple words, if

$$\begin{vmatrix} (d_{11}-\mu) & d_{12} d_{13} & \dots\dots & \dots . d_{1n} \\ d_{21} & (d_{22}-\mu) & d_{23} \quad \dots & \dots . d_{2n} \\ \dots\dots\dots . & \dots\dots\dots . & & \dots\dots\dots \\ d_{n1} & d_{n2} & d_{n3} \dots\dots & \dots . (d_{nn}-\mu) \end{vmatrix} = 0,$$

then on simplifying this equation, we get an equation inμ with degree n. We call this n^{th} degree equation asthe characteristic equation of the matrix D and its roots as Eigenvalues.The solutionof the equation obtained is a column matrix T = $\begin{bmatrix} t_1 \\ t_2 \\ \dots \\ t_n \end{bmatrix}$corresponding to each Eigenvalue. We call this column matrix T as Eigenvector.

Formulation:

To derive the governing differential equation:

We will take a series Ł - C- Ɍ network to which a steady excitation voltage source of constant potential V is applied through a key K as shown in figure 1. As the switch is closed at t = 0, the potential drops across the network elements are given by

$V_R\,(t) = I(t)Ɍ$, $V_L\,(t) = ŁĐ_t[I(t)]$ and $V_C\,(t) = \frac{q(t)}{C}$, where q (t) is the charge on the capacitor at any instant of time t.

The application of Kirchhoff's loop law to the loop as the switch is closed at the instant t = 0 provides

$V_R(t) + V_Ł\,(t) + V_C\,(t) = V$

Or I (t) Ɍ+ $ŁĐ_t[I(t)] + \frac{q(t)}{C} = V$ ….. (1) $Đ_t \equiv \frac{d}{dt}$.

When we differentiate equation (1), we get a linear homogeneous differential equation of order 2 as given below:

$ɌĐ_t[I(t)] + ŁĐ_t^2[I(t)] + \frac{1}{C}I(t) = 0$, where$I(t) = Đ_t[q(t)]$ is the instantaneous electric current flowing in the series Ł - C- Ɍnetwork circuit.

Or $ŁĐ_t^2[I(t)] + ɌĐ_t[I(t)] + \frac{1}{C}I(t) = 0$

Or $Đ_t^2[I(t)] + \frac{Ɍ}{Ł}Đ_t[I(t)] + \frac{1}{ŁC}\, I(t) = 0$…. (2)

To solve the governing differential equation:

To solve equation (2), we first write the relevant boundary conditions as follows:

(i) Since the current through the inductor and the electric potential across the capacitor cannot be changed instantaneously, therefore, as the switch is closed at the instant t = 0, then I (0) = 0.

(ii) Since at the instant t = 0, I (0) = 0, therefore, equation (1) provides $ŁĐ_t[I(0)] = V$ Or $Đ_t[I(0)] = \frac{V}{Ł}$.

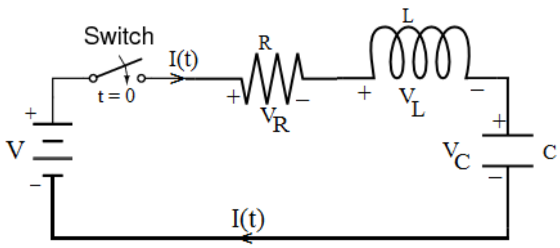

Figure 1: Series R-L-C network with steady voltage source.

Further, let us put

$$I(t) = I_1(t) \ldots\ldots\ldots (3)$$

And
$$Đ_t[I_1(t)] = I_2(t) \ldots\ldots. (4)$$

We can rewrite equation (2) as

$$Đ_t[I_2(t)] + \frac{R}{Ł} I_2(t) + \frac{1}{ŁC} I_1(t) = 0$$

Or
$$Đ_t[I_2(t)] = -\frac{1}{ŁC} I_1(t) - \frac{R}{Ł} I_2(t) \ldots. (5)$$

We can write the differential equations (4) and (5) ina single matrix formas

$$Đ_t \begin{bmatrix} I_1(t) \\ I_2(t) \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -\frac{1}{ŁC} & -\frac{R}{Ł} \end{bmatrix} \begin{bmatrix} I_1(t) \\ I_2(t) \end{bmatrix}$$

Equating the determinant of $\begin{bmatrix} 0 & 1 \\ -\frac{1}{ŁC} & -\frac{R}{Ł} \end{bmatrix}$ to zero, weobtain its characteristic equation as

$$\begin{vmatrix} 0 - \mu & 1 \\ -\frac{1}{ŁC} & -\frac{R}{Ł} - \mu \end{vmatrix} = 0$$

On expanding the determinant, we get

$$\mu^2 + \frac{R}{Ł} \mu + \frac{1}{ŁC} = 0 \ldots\ldots. (6)$$

This equation (6) is quadratic in μ and its roots are given by

$$\mu = \frac{-\frac{R}{Ł} \pm \sqrt{\left(\frac{R}{Ł}\right)^2 - \frac{4}{ŁC}}}{2}$$

Or
$$\mu = -\frac{R}{2Ł} \pm \frac{1}{2Ł}\sqrt{R^2 - \frac{4Ł}{C}}$$

Therefore, the roots of the equation (6) are $\mu_1 = -\frac{R}{2Ł} + \frac{1}{2Ł}\sqrt{R^2 - \frac{4Ł}{C}} \ldots\ldots. (7)$

And
$$\mu_2 = -\frac{R}{2Ł} - \frac{1}{2Ł}\sqrt{R^2 - \frac{4Ł}{C}} \ldots\ldots\ldots.. (8)$$

The product of roots is obtained by multiplying equation (7) and (8) i.e.

$$\mu_1\mu_2 = \left(-\frac{R}{2Ł} + \frac{1}{2Ł}\sqrt{R^2 - \frac{4Ł}{C}}\right)\left(-\frac{R}{2Ł} - \frac{1}{2Ł}\sqrt{R^2 - \frac{4Ł}{C}}\right)$$

On simplifying the product of terms on right hand side of this equation, we get

$\mu_1\mu_2 = \frac{1}{ŁC} \ldots\ldots. (9)$

To determine Eigenvectors:

The Eigenvector for the root $\mu = \mu_1 = -\frac{R}{2Ł} + \frac{1}{2Ł}\sqrt{R^2 - \frac{4Ł}{C}}$ is given by

$$\begin{bmatrix} 0 - \mu_1 & 1 \\ -\frac{1}{ŁC} & -\frac{R}{Ł} - \mu_1 \end{bmatrix}\begin{bmatrix} t_1 \\ t_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

This results

$$-\mu_1 t_1 + t_2 = 0 \text{ .... } (10)$$

And

$$-\frac{1}{ŁC}t_1 - \left(\frac{R}{Ł} + \mu_1\right)t_2 = 0 \text{ ... ... } (11)$$

Solving equations (10) and (11), we can write

$$\begin{bmatrix} t_1 \\ t_2 \end{bmatrix} = \begin{bmatrix} \mu_1 + \frac{R}{Ł} + 1 \\ \mu_1 - \frac{1}{ŁC} \end{bmatrix}$$

And the Eigenvector for the root $\mu = \mu_2 = -\frac{R}{2Ł} - \frac{1}{2Ł}\sqrt{R^2 - \frac{4Ł}{C}}$ is given by

$$\begin{bmatrix} 0 - \mu_2 & 1 \\ -\frac{1}{ŁC} & -\frac{R}{Ł} - \mu_2 \end{bmatrix}\begin{bmatrix} t_1 \\ t_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

This results

$$-\mu_2 t_1 + t_2 = 0 \text{ .... } (12)$$

And

$$-\frac{1}{ŁC}t_1 - \left(\frac{R}{Ł} + \mu_2\right)t_2 = 0 \text{ ... ... } (13)$$

Solving equations (12) and (13), we can write

$$\begin{bmatrix} t_1 \\ t_2 \end{bmatrix} = \begin{bmatrix} \mu_2 + \frac{R}{Ł} + 1 \\ \mu_2 - \frac{1}{ŁC} \end{bmatrix}$$

The matrix of Eigenvectors is $\begin{bmatrix} \mu_1 + \frac{R}{Ł} + 1 & \mu_2 + \frac{R}{Ł} + 1 \\ \mu_1 - \frac{1}{ŁC} & \mu_2 - \frac{1}{ŁC} \end{bmatrix}$.

Let P $= \begin{bmatrix} \mu_1 + \frac{R}{Ł} + 1 & \mu_2 + \frac{R}{Ł} + 1 \\ \mu_1 - \frac{1}{ŁC} & \mu_2 - \frac{1}{ŁC} \end{bmatrix}$, then the determinant of P i.e. $|P|$ can be written as

$|P| = \begin{vmatrix} \mu_1 + \frac{R}{Ł} + 1 & \mu_2 + \frac{R}{Ł} + 1 \\ \mu_1 - \frac{1}{ŁC} & \mu_2 - \frac{1}{ŁC} \end{vmatrix}$

On expanding the determinant and using equation (9), we obtain

$|P| = (\mu_2 - \mu_1)(\frac{1}{ŁC} + \frac{R}{Ł} + 1)$

The inverse of P which is also a matrix can be written as

$$P^{-1}=\frac{1}{(\mu_2-\mu_1)(\frac{1}{ŁC}+\frac{Ɍ}{Ł}+1)}\begin{bmatrix}\mu_2-\frac{1}{ŁC} & -(\mu_2+\frac{Ɍ}{Ł}+1)\\ -(\mu_1-\frac{1}{ŁC}) & \mu_1+\frac{Ɍ}{Ł}+1\end{bmatrix}$$

To Determine $Pe^{\mu t}P^{-1}$:

$$Pe^{\mu t}P^{-1}=$$

$$\begin{bmatrix}\mu_1+\frac{Ɍ}{Ł}+1 & \mu_2+\frac{Ɍ}{Ł}+1\\ \mu_1-\frac{1}{ŁC} & \mu_2-\frac{1}{ŁC}\end{bmatrix}\begin{bmatrix}\exp(\mu_1\,t) & 0\\ 0 & \exp(\mu_2\,t)\end{bmatrix}\frac{1}{(\mu_2-\mu_1)(\frac{1}{ŁC}+\frac{Ɍ}{Ł}+1)}\begin{bmatrix}\mu_2-\frac{1}{ŁC} & -(\mu_2+\frac{Ɍ}{Ł}+1)\\ -(\mu_1-\frac{1}{ŁC}) & \mu_1+\frac{Ɍ}{Ł}+1\end{bmatrix}$$

$$=\frac{1}{(\mu_2-\mu_1)(\frac{1}{ŁC}+\frac{Ɍ}{Ł}+1)}\times$$

$$\begin{bmatrix}(\mu_1+\frac{Ɍ}{Ł}+1)\exp(\mu_1\,t) & (\mu_2+\frac{Ɍ}{Ł}+1)\exp(\mu_2\,t)\\ (\mu_1-\frac{1}{ŁC})\exp(\mu_1\,t) & (\mu_2-\frac{1}{ŁC})\exp(\mu_2\,t)\end{bmatrix}\begin{bmatrix}\mu_2-\frac{1}{ŁC} & -(\mu_2+\frac{Ɍ}{Ł}+1)\\ -(\mu_1-\frac{1}{ŁC}) & \mu_1+\frac{Ɍ}{Ł}+1\end{bmatrix}$$

$$=\frac{1}{(\mu_2-\mu_1)(\frac{1}{ŁC}+\frac{Ɍ}{Ł}+1)}\times$$

$$\begin{bmatrix}(\mu_1+\frac{Ɍ}{Ł}+1)(\mu_2-\frac{1}{ŁC})\exp(\mu_1\,t)-(\mu_1-\frac{1}{ŁC})(\mu_2+\frac{Ɍ}{Ł}+1)\exp(\mu_2\,t) & (\mu_1+\frac{Ɍ}{Ł}+1)\,(\mu_2+\frac{Ɍ}{Ł}+1)[\exp(\mu_2\,t)-\exp(\mu_1\,t)]\\ (\mu_2-\frac{1}{ŁC})(\mu_1-\frac{1}{ŁC})\,[\exp(\mu_1\,t)-\exp(\mu_2\,t)] & -(\mu_2+\frac{Ɍ}{Ł}+1)(\mu_1-\frac{1}{ŁC})\exp(\mu_1\,t)+(\mu_1+\frac{Ɍ}{Ł}+1)(\mu_2-\frac{1}{ŁC})\exp(\mu_2\,t\end{bmatrix}$$

The application of initial boundary conditions: $I_1(0)=I(0)=0$ and $I_2(0)=Đ_t[I(0)]=\frac{V}{Ł}$ provides

$$\begin{bmatrix}I_1(t)\\ I_2(t)\end{bmatrix}=\frac{1}{(\mu_2-\mu_1)\left(\frac{1}{ŁC}+\frac{Ɍ}{Ł}+1\right)}\times$$

$$\begin{bmatrix}(\mu_1+\frac{Ɍ}{Ł}+1)(\mu_2-\frac{1}{ŁC})\exp(\mu_1\,t)-(\mu_1-\frac{1}{ŁC})(\mu_2+\frac{Ɍ}{Ł}+1)\exp(\mu_2\,t) & (\mu_1+\frac{Ɍ}{Ł}+1)\,(\mu_2+\frac{Ɍ}{Ł}+1)[\exp(\mu_2\,t)-\exp(\mu_1\,t)]\\ (\mu_2-\frac{1}{ŁC})(\mu_1-\frac{1}{ŁC})\,[\exp(\mu_1\,t)-\exp(\mu_2\,t)] & -(\mu_2+\frac{Ɍ}{Ł}+1)(\mu_1-\frac{1}{ŁC})\exp(\mu_1\,t)+(\mu_1+\frac{Ɍ}{Ł}+1)(\mu_2-\frac{1}{ŁC})\exp(\mu_2\,t\end{bmatrix}\begin{bmatrix}0\\ \frac{V}{Ł}\end{bmatrix}$$

Or

$$\begin{bmatrix} I_1(t) \\ I_2(t) \end{bmatrix} = \frac{1}{(\mu_2 - \mu_1)\left(\frac{1}{ŁC}+\frac{R}{Ł}+1\right)} \begin{bmatrix} \frac{V}{L}(\mu_1 + \frac{R}{Ł} + 1)\ (\mu_2 + \frac{R}{Ł} + 1)[\exp(\mu_2\ t) - \exp(\mu_1\ t)] \\ \frac{V}{L}[-(\mu_2 + \frac{R}{Ł} + 1)(\mu_1 - \frac{1}{ŁC})\exp(\mu_1\ t) + (\mu_1 + \frac{R}{Ł} + 1)(\mu_2 - \frac{1}{ŁC})\exp(\mu_2\ t)] \end{bmatrix}$$

This results

$$I_1(t) = \frac{\frac{V}{Ł}(\mu_1 + \frac{R}{Ł} + 1)\ (\mu_2 + \frac{R}{Ł} + 1)[\exp(\mu_2\ t) - \exp(\mu_1\ t)]}{(\mu_2 - \mu_1)\left(\frac{1}{ŁC} + \frac{R}{Ł} + 1\right)}$$

Or

$$I(t) = \frac{\frac{V}{Ł}(\mu_1 + \frac{R}{Ł} + 1)\ (\mu_2 + \frac{R}{Ł} + 1)[\exp(\mu_2\ t) - \exp(\mu_1\ t)]}{(\mu_2 - \mu_1)\left(\frac{1}{ŁC} + \frac{R}{Ł} + 1\right)}$$

Or

$$I(t) = \frac{\frac{V}{Ł}(\mu_1 + \frac{R}{Ł} + 1)\ (\mu_2 + \frac{R}{Ł} + 1)\ [\exp(\mu_1\ t) - \exp(\mu_2\ t)]}{(\mu_1 - \mu_2)\left(\frac{1}{ŁC} + \frac{R}{Ł} + 1\right)} \ldots\ldots\ldots\ldots (14)$$

And

$$I_2(t) = \frac{\frac{V}{Ł}[-(\mu_2 + \frac{R}{Ł} + 1)(\mu_1 - \frac{1}{ŁC})\exp(\mu_1\ t) + (\mu_1 + \frac{R}{Ł} + 1)(\mu_2 - \frac{1}{ŁC})\exp(\mu_2\ t)]}{(\mu_2 - \mu_1)\left(\frac{1}{ŁC} + \frac{R}{Ł} + 1\right)}$$

Or

$$Đ_t[I(t)] = \frac{\frac{V}{Ł}[-(\mu_2 + \frac{R}{Ł} + 1)\ (\mu_1 - \frac{1}{ŁC})\exp(\mu_1\ t)\ +\ (\mu_1 + \frac{R}{Ł} + 1)\ (\mu_2 - \frac{1}{ŁC})\exp(\mu_2\ t)]}{(\mu_2 - \mu_1)\left(\frac{1}{ŁC} + \frac{R}{Ł} + 1\right)}$$

Or

$$Đ_t[I(t)] = \frac{\frac{V}{Ł}[(\mu_2 + \frac{R}{Ł} + 1)\ (\mu_1 - \frac{1}{ŁC})\ exp(\mu_1 t) - (\mu_1 + \frac{R}{Ł} + 1)\ (\mu_2 - \frac{1}{ŁC})\ exp(\mu_2 t)]}{(\mu_1 - \mu_2)\left(\frac{1}{ŁC} + \frac{R}{Ł} + 1\right)} \ldots\ldots (15)$$

Using equations (7) and (8), we can obtain

$$(\mu_1 + \frac{R}{Ł} + 1)\ (\mu_2 + \frac{R}{Ł} + 1) = \left(-\frac{R}{2Ł} + \frac{1}{2Ł}\sqrt{R^2 - \frac{4Ł}{C}} + \frac{R}{Ł} + 1\right)\left(-\frac{R}{2Ł} - \frac{1}{2Ł}\sqrt{R^2 - \frac{4Ł}{C}} + \frac{R}{Ł} + 1\right)$$

Or

$$(\mu_1 + \frac{R}{Ł} + 1)\ (\mu_2 + \frac{R}{Ł} + 1) = \left(1 + \frac{R}{Ł} + \frac{1}{2Ł}\sqrt{R^2 - \frac{4Ł}{C}}\right)\left(1 + \frac{R}{Ł} - \frac{1}{2Ł}\sqrt{R^2 - \frac{4Ł}{C}}\right)$$

Or

$$(\mu_1 + \frac{Ɽ}{Ł} + 1)\,(\mu_2 + \frac{Ɽ}{Ł} + 1) = \left(1 + \frac{Ɽ}{Ł}\right)^2 - \left(\frac{1}{2Ł}\sqrt{Ɽ^2 - \frac{4Ł}{C}}\right)^2$$

On simplification of the right-hand side of this equation, we obtain

$$(\mu_1 + \frac{Ɽ}{Ł} + 1)\,(\mu_2 + \frac{Ɽ}{Ł} + 1) = \left(\frac{1}{ŁC} + \frac{Ɽ}{Ł} + 1\right) \ldots\ldots\ldots..(16)$$

And

$$(\mu_1 - \mu_2) = -\frac{Ɽ}{2Ł} + \frac{1}{2Ł}\sqrt{Ɽ^2 - \frac{4Ł}{C}} - \left(-\frac{Ɽ}{2Ł} - \frac{1}{2Ł}\sqrt{Ɽ^2 - \frac{4Ł}{C}}\right)$$

On simplification of the right-hand side of this equation, we obtain

$$(\mu_1 - \mu_2) = \frac{1}{Ł}\sqrt{Ɽ^2 - \frac{4Ł}{C}} \ldots\ldots\ldots (17)$$

Using equations(16) and (17) in equation (14), we get

$$I(t) = \frac{\frac{V}{Ł}[exp(\mu_1 t) - exp(\mu_2 t)]}{\frac{1}{Ł}\sqrt{Ɽ^2 - \frac{4Ł}{C}}}$$

Or

$$I(t) = \frac{V[exp(\mu_1 t) - exp(\mu_2 t)]}{\sqrt{Ɽ^2 - \frac{4Ł}{C}}} \ldots\ldots\ldots (18)$$

Substituting equations (7) and (8) in equation (18), we get

$$I(t) = \frac{V\left\{exp\left[\left(-\frac{Ɽ}{2Ł} + \frac{1}{2Ł}\sqrt{Ɽ^2 - \frac{4Ł}{C}}\right)t\right] - exp\left[\left(-\frac{Ɽ}{2Ł} - \frac{1}{2Ł}\sqrt{Ɽ^2 - \frac{4Ł}{C}}\right)t\right]\right\}}{\sqrt{Ɽ^2 - \frac{4Ł}{C}}}$$

Or

$$\mathrm{I(t)} = \frac{\mathrm{V}\exp(-\frac{Ɽ}{2Ł}t)\left\{\exp\left[\left(\frac{1}{2Ł}\sqrt{Ɽ^2 - \frac{4Ł}{C}}\right)t\right] - \exp\left[\left(-\frac{1}{2Ł}\sqrt{Ɽ^2 - \frac{4Ł}{C}}\right)t\right]\right\}}{\sqrt{Ɽ^2 - \frac{4Ł}{C}}} \ldots\ldots\ldots\ldots (19)$$

This equation (19) provides an expression for the electric current flowing in a series $Ł - C - Ɽ$ network connected to an excitation source of constant potential V. It reveals that the presence of an inductor and a capacitor in the series $Ł - C - Ɽ$ network leads to variation in the flow of current even if the excitation source connected to the network has a constant value of potential. It is clear from the equation that the nature of current in the series $Ł - C - Ɽ$ network depends on the nature of the quantity $\sqrt{Ɽ^2 - \frac{4Ł}{C}}$, whether it is real, zero or imaginary. The nature of the quantity $\sqrt{Ɽ^2 - \frac{4Ł}{C}}$, in turn, depends on the values of $Ɽ^2 and \frac{4Ł}{C}$. We have the following three possibilities:

Possibility I: If the values of network elements Ł, C *and* Rare so chosen that R>$\left(\frac{4Ł}{C}\right)^{\frac{1}{2}}$, then the quantity $\sqrt{R^2-\frac{4Ł}{C}}$ is real. In such a case, equation (19) can be rewritten as

$$I(t)=\frac{2V exp(-\frac{R}{2Ł}t)\, sinh\left(\frac{1}{2Ł}\sqrt{R^2-\frac{4Ł}{C}}\right)t}{\sqrt{R^2-\frac{4Ł}{C}}} \quad \ldots\ldots\ldots\ldots.. (20)$$

This equation (20) reveals that the current in the series Ł − C − R network is non – oscillatory as $sinh\left(\frac{1}{2Ł}\sqrt{R^2-\frac{4Ł}{C}}\right)t$ is non-periodic function of time, and it decays gradually to zero.

Possibility II: If the values of network elements Ł, C *and* R are so chosen that R =$\left(\frac{4Ł}{C}\right)^{\frac{1}{2}}$, then the quantity $\sqrt{R^2-\frac{4Ł}{C}}$ is zero. In such a case, equation (19) reveals that the current in the series Ł − C − R network is indeterminate, which is impossible. If the quantity $\sqrt{R^2-\frac{4Ł}{C}}$ is so small that it approaches to zero, then on expanding the exponential terms containing the quantity $\sqrt{R^2-\frac{4Ł}{C}}$ and taking only the first two terms, we can rewrite equation (19) as

$$I(t)=\frac{V\,exp\left(-\frac{R}{2Ł}t\right)\left\{1+\left(\frac{1}{2Ł}\sqrt{R^2-\frac{4Ł}{C}}\right)t-[\,1-\left(\frac{1}{2Ł}\sqrt{R^2-\frac{4Ł}{C}}\right)t]\right\}}{\sqrt{R^2-\frac{4Ł}{C}}}$$

Or $$I(t)=\frac{V}{L}texp\left(-\frac{R}{2Ł}t\right) \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots.. (21)$$

This equation (21) reveals that the current in the series Ł − C − R network is non – oscillatory and it decays to zero in the least possible time.

Possibility III: If the values of network elements Ł, C *and* R are so chosen that R<$\left(\frac{4Ł}{C}\right)^{\frac{1}{2}}$, then the quantity $\sqrt{R^2-\frac{4Ł}{C}}$ is imaginary. We can write the quantity $\sqrt{R^2-\frac{4Ł}{C}}$ as

$$\sqrt{R^2-\frac{4Ł}{C}}=i\sqrt{\frac{4Ł}{C}-R^2}\ldots\ldots\ldots\ldots.. (22)$$

Using equation (22), we can rewrite equation (19) as

$$I(t)=\frac{Vexp(-\frac{R}{2Ł}t)\left\{exp\left[\left(\frac{1}{2Ł}i\sqrt{\frac{4Ł}{C}-R^2}\right)t\right]-exp\left[\left(-\frac{1}{2Ł}i\sqrt{\frac{4Ł}{C}-R^2}\right)t\right]\right\}}{i\sqrt{\frac{4Ł}{C}-R^2}}$$

Or $$I(t)=\frac{2V\exp\left(-\frac{R}{2Ł}t\right)\sin\left[\left(\frac{1}{2Ł}\sqrt{\frac{4Ł}{C}-R^2}\right)t\right]}{\sqrt{\frac{4Ł}{C}-R^2}} \quad \ldots\ldots\ldots\ldots.. (23)$$

This equation (23) reveals that the current in the series $Ł - C - R$ network is oscillatory with amplitude $\frac{2V\,exp\left(-\frac{R}{2Ł}t\right)}{\sqrt{\frac{4Ł}{C} - R^2}}$ which decreases exponentially with time, and oscillating frequency $\frac{1}{4\pi Ł}\sqrt{\frac{4Ł}{C} - R^2}$.

Conclusions:

In this paper we have obtained the response (i.e. current) of a series $Ł - C - R$ network connected to an excitation voltage source of constant potential by matrix method. We concluded that the response (i.e. current) can be oscillatory or non-oscillatory depending on the values of elements $Ł, C$ and $R$of the network. The nature of current can be made oscillatory by selecting the values of network elements$Ł, C$ and$R$such that $R<\left(\frac{4Ł}{C}\right)^{\frac{1}{2}}$, andin such a condition the oscillating frequencyis independent of excitation voltagesource of constant potential.

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