A New Approach for Solving Fully Fuzzy Linear Programming Using the Centroid of Centroid Index Method

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Abstract

The fuzzy linear programming (FLP) problem is applied in disciplanes of engineering, business and sciences.In this paper, a new ranking centroid of centroid index method for generalized LR fuzzy numbers is proposed to overcome the drawbacks of most of the existing methods and it is applied to solve the FLP problem, in which where all the the constraints are fuzzy inequality or equality, and coefficients and decision variables are generalized LR fuzzy numbers. The most important characteristic of the proposed fuzzy ranking method is that, it takes into deliberation the decision makers positive approach as well as the index of modality. While ranking the fuzzy numbers where the discrimination is not possible it notify the importance of spreads or central value. The proposed ranking method is explained with different examples existing in the literature and a comparative study is made. The proposed FLP model is exemplified by numerical example.

Keywords: *Fuzzy linear programming, LR fuzzy number, Centroid of Centroid ranking index.*

1. Introduction

Since six decades has passed from its first description and clarification, Linear programming *(LP)* is still useful for promoting a new approach for blending real-world problems in the outline of linear programming. Linear programming *(LP)* or linear optimization is one of most practical techniques in operation research, which finds the best extractable solution with respect to the constraints. In real-world problems, values of the parameters in *LP* problem should be precisely described and evaluated. However, in real-world applications, the parameters are often illusive. The optimal solution of *LP* depends on a limited number of constraints; therefore, much of the collected information has a little impact on the solution. So, fuzzy linear programming *(FLP)* has been developed for treating uncertainty in the setting of optimization problems and various attempts have been made to study the solution of *FLP* problems, either from theoretical or computational point of view. It is useful to consider the knowledge of experts about the parameters as fuzzy data. The concept of decision in fuzzy environment was first proposed by Bellman and Zadeh [1]. Tanaka et al. [2] proposed a new method for solving the fuzzy mathematical programming problem. First the formulation of fuzzy linear programming was proposed by Zimmermann [3] .A method for solving *LP* problem with uncertain constraints using ranking function was proposed by Maleki [4]. For solving *LP* problem with fuzzy coefficients in objective function was proposed by Zhang et al. [5]. Jimenez et al. [6] proposed a method for solving linear programing problems with all the coefficients as fuzzy numbers and used fuzzy ranking method to rank the fuzzy objective values and to deal with inequality relation on constraints and it allowed them to work with the concept of feasibility degree. Based on a kind of defuzzification method Allahviranloo et al. [7] brought up a method for solving fully fuzzy *LP* problem. *FLP* problem with fuzzy parameters using complementary slackness property was solved by Ebrahimnejad and Nasseri [8]. Some practical methods to solve a fully fuzzy linear system which are comparable to the well-known methods was proposed by Dehghan et al. [9] and then they extended a new method employing linear programming *(LP)* for solving square and non square fuzzy systems. Lotfi et al. [10] applied the concept of the symmetric triangular fuzzy number and introduced an approach to defuzzify a general fuzzy quantity, with the special ranking on fuzzy numbers, the fully fuzzy linear programming problem transformed into multi objective linear programming problem where all variables and parameters are crisp. Kumar et al. [11] pointed out the shortcomings of the methods of [9, 10], to overcome these shortcomings, they proposed a new method for finding the fuzzy optimal solution of fully *FLP* problems with equality constraints. This method also had shortcomings which were corrected by Saberi Najafi and Edalatpanah [12]. By using *LR* fuzzy numbers and ranking function Shamooshaki et al. [13] established a new scheme for fully *FLP* . In recent years, ranking fuzzy numbers is an important tool in linguistic decision making and some other fuzzy application systems. In fuzzy decision analysis, fuzzy quantities are used to describe the performance of alternatives in modelling a real-world problem. Several ranking procedures were proposed since 1976, in literature some of the existing ranking methods cannot discriminate fuzzy quantities and some are counter-intuitive results in certain cases. The fuzzy ranking methods like Chen and Sanguansat [14], Xu *et al.* [15], Cheng [16], Chu and Tsao [17] and Yagger [18], have some drawbacks for ranking *LR* fuzzy number, i.e., they cannot discriminate fuzzy numbers in numerous cases and some methods do not agree with human approach, some of the existing methods cannot rank their images, they are unable to distinguish fuzzy numbers with different heights, and with different signs, are too pessimistic, whereas, some methods cannot rank crisp numbers which are a special case of fuzzy numbers.

In this paper, a new method for ranking generalized *LR* fuzzy numbers is presented which can overcome the drawbacks of the existing methods. The method of defuzzification is applied by taking a ranking function which maps each fuzzy number to a real number which is the distance between the centroid point of that fuzzy number and the origin. A number of ranking procedures proposed in literature use centroid of trapezoid as reference point to rank fuzzy numbers as the centroid is the balancing point of the trapezoid. But the centroid of centroids can be considered a much more balancing point than the centroid. Further, this method uses an index of optimism to reflect the decision maker's optimistic attitude and also uses an index of modality that represents the importance of mode and spreads where the distance between the centroid point and the origin are equal for fuzzy numbers. This new fuzzy ranking method is then applied to fuzzy linear programming. The proposed fuzzy linear programming methodology provides a useful way to deal with fuzzy linear programming problems based on the generalized *LR* fuzzy numbers. However some of the existing fuzzy linear programming models are restricted only to normal *LR* fuzzy numbers. But the proposed fuzzy ranking method and fuzzy linear programming problem can take generalized *LR* fuzzy numbers as well as normal *LR* fuzzy numbers.

The rest of the paper is organised as follows, Section 2 gives basic concepts of *LR* fuzzy numbers and operations on *LR* fuzzy numbers. In Section 3, the proposed ranking method, working rule, and explanation of some numerical examples from the existing methods and a comparative study is given. The mathematical formulation of the proposed fuzzy linear programming method and it's working rule is given in Section 4. A numerical example to illustrate the fuzzy linear programming method is given in Section 5. Conclusion is given in Section 6.

2. Basic Concepts

In this section, *LR* fuzzy number and operations on *LR* fuzzy number are reviewed from Dubois and Prade [19]

A fuzzy number $\tilde{A} = (m,n,\alpha,\beta)_{LR}$ is said to be an *LR* fuzzy number if

$$\mu_{\tilde{A}}(x) = \begin{cases} L\left(\frac{m-x}{\alpha}\right), & x \le m, \alpha > 0, \\ R\left(\frac{x-n}{\beta}\right), & x \ge n, \beta > 0, \\ 1, & otherwise. \end{cases}$$

If $m = n$ then $\tilde{A} = (m,n,\alpha,\beta)_{LR}$ will be converted into $\tilde{A} = (m,\alpha,\beta)_{LR}$ and is said to be a triangular *LR* fuzzy number. *L* and *R* are called left and right reference functions, which are continuous, non-increasing functions that define the left and right shapes of $\mu_{\tilde{A}}(x)$ respectively and $L(0) = R(0) = 1$.

Arithmetic Operation on *LR* fuzzy number

For two generalized *LR* fuzzy numbers $\tilde{A}_1$ and $\tilde{A}_2$ where $\tilde{A}_1 = (m_1, n_1, \alpha_1, \beta_1; w_1)_{LR}$ and $\tilde{A}_2 = (m_2, n_2, \alpha_2, \beta_2; w_2)_{LR}$, is defined as:

$$\tilde{A}_1 \oplus \tilde{A}_2 = (m_1 + m_2, n_1 + n_2, \alpha_1 + \alpha_2, \beta_1 + \beta_2; \min(w_1, w_2))_{LR}$$

3. New ranking approach of *LR* fuzzy numbers with a distance method using Centroid of Centroids and an Index of Optimism and Modality.

In this Section, a new ranking approach is given for the ranking of *LR* fuzzzy numbers, to bring out strong perceptive ability in a ranking procedure, a method which can classify fuzzy numbers in an effective manner is given. As the index of optimism alone is not sufficient to discriminate fuzzy numbers, in this method, besides the coordinates of the centroid of centroids of generalized trapezoidal fuzzy numbers, an index of optimism which reflects a decision maker optimistic attitude towards ordering fuzzy numbers along with an index of modality which show the importance of central value and spreads of fuzzy numbers is used.

3.1 New Ranking Method

Generally,in trapezoid the Centroid is considered as the balancing point. In this new ranking method the trapezoid *(APQD)* is divided into the triangle *(APB)*, the rectangle *(BPQC)* and the triangle *(CQD)*, and the centroids of these three figures *APB*, *BPQC* and *CQD* be G_1, G_2 and G_3 respectively as shown in Figure 1.

The centroid of the triangle *(APB)*, where $A = (m-\alpha, 0)$, $P = (m, w)$, $C = (n, 0)$ is,

$$G_1 = \left(\frac{3m-\alpha}{3} , \frac{w}{3} \right)$$

The centroid of the triangle *(QCD)*, where $Q = (n, w)$, $C = (n, 0)$, $D = (n+\beta, 0)$ is,

$$G_2 = \left(\frac{3n+\beta}{3} , \frac{w}{3} \right)$$

The centroid of the rectangle *(BPQC)*, where $B = (m, 0)$, $P = (m, w)$, $Q = (n, w)$, $C = (n, 0)$, is,

$$G_3 = \left(\frac{m+n}{2} , \frac{w}{2} \right)$$

Equation of the line $G_1 G_3$ is $y = \frac{w}{3}$ and G_2 does not lie on the line $G_1 G_3$. Therefore G_1, G_2 and G_3 are non-collinear and they form a triangle.

The Centroid of the triangle $(G_1G_2G_3)$ is the intersection of the three centroid points G_1, G_2 and G_3 as shown in Figure1. Since each Centroid point is a balancing point of each individual polygon, therefore Centroid of the triangle $(G_1G_2G_3)$ is taken as the better point of reference than the centroid point of the trapezoid, as it is much more balancing point, it is used to define the ranking of generalized *LR* fuzzy numbers $\tilde{A} = (m, n, \alpha, \beta; w)_{LR}$.

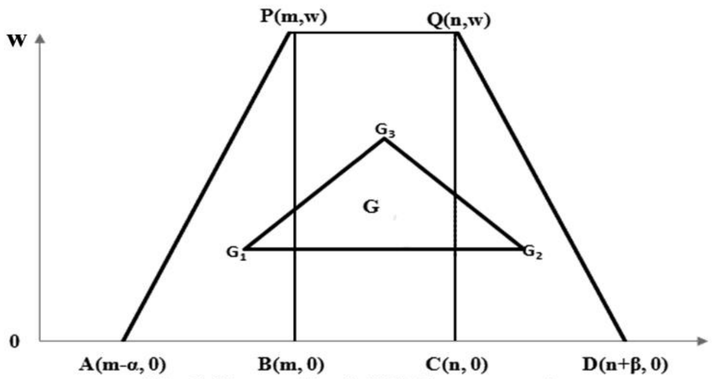


Fig.1 Generalized LR fuzzy number

The Centroid $G_{\tilde{A}}(x,y)$ of the triangle $(G_1G_2G_3)$ with vertices G_1, G_2 and G_3 for the generalized trapezoidal LR fuzzy number $\tilde{A} = (m,n,\alpha,\beta;\, w)_{LR}$ is

$$G_{\tilde{A}}(x,y) = \left(\frac{9m+9n-2\alpha+2\beta}{18} \,,\, \frac{7w}{18} \right) \tag{1}$$

The Centroid $G_{\tilde{A}}(x,y)$ of the triangle $(G_1G_2G_3)$ with vertices G_1, G_2 and G_3 for the generalized triangular LR fuzzy number $\tilde{A} = (m,n,\alpha,\beta;\, w)_{LR}$ i.e., $m = n$ is

$$G_{\tilde{A}}(x,y) = \left(\frac{18m-2\alpha+2\beta}{18} \,,\, \frac{7w}{18} \right) \tag{2}$$

The ranking function of LR fuzzy number $\tilde{A} = (m,n,\alpha,\beta;w)$.which maps the set of all fuzzy numbers to a set of real numbers is defined as the distance between centroid of the centroids of the generalized LR fuzzy number $\tilde{A}$ and the original point is,

$$R_{\tilde{A}} = \sqrt{(x)^2 + (y)^2} \tag{3}$$

3.1.1 Mode and Spreads

The mode and spreads for a generalized LR fuzzy number $\tilde{A} = (m,n,\alpha,\beta;w)$ are defined as

Mode or Central value of

$$\tilde{A} = M(\tilde{A}) = \frac{m+n}{2} \tag{4}$$

Total Spread of

$$\tilde{A} = TS\left(\tilde{A}\right) = n + \beta - m + \alpha \tag{5}$$

Left Spread of

$$\tilde{A} = LS\left(\tilde{A}\right) = \alpha \tag{6}$$

Right Spread of

$$\tilde{A} = RS\left(\tilde{A}\right) = \beta \tag{7}$$

3.1.2 Index using Centroid of Centroids and Optimism

For a generalized *LR* fuzzy number $\tilde{A} = (m,n,\alpha,\beta;w).$, with centroid of centroids $G_{\tilde{A}}(x,y)$ defined by Eq. (1), the index associated with the ranking is

$$I_{\gamma}\left(\tilde{A}\right) = \gamma\, y + (1-\gamma)x \tag{8}$$

where $\gamma \in [0,1]$is the index of optimism which represents the degree of optimism of a decision maker.

- **(i)** when $\gamma = 0$ the pessimistic decision maker's view point is find, which is equal to the distance of the centroid from *y*-axis.
- **(ii)** when $\gamma = 1$ an optimistic decision maker's view point is find and is equal to the distance of the centroid from *x*-axis, and
- **(iii)** when $\gamma = 0.5$ the moderate decision maker's view point is find and is equal to the mean of the distances of centroid from *y* and *x*- axes.

The larger the value of γ is the higher the degree of optimism of the decision maker. The index of optimism is not alone sufficient to discriminate fuzzy numbers as this uses only the extreme values of the centroid of centroids. Hence, by using an index of modality which represents the importance of central value along with index of optimism is improved.

3.1.3 Indexes using Centroid of Centroids, Optimism and Modality

For a generalized *LR* fuzzy number $\tilde{A} = (m,n,\alpha,\beta;w).$, with centroid of centroids $G_{\tilde{A}}(x,y)$ defined by Eq. (1), the following indexes are defined which combines the index of optimism γ and index of modality η that can be allied with the ranking.

$$I^{1}_{\eta,\gamma}\left(\tilde{A}\right) = \eta\, M\left(\tilde{A}\right) + (1-\eta)\, I_{\gamma}\left(\tilde{A}\right) \tag{9}$$

$$I^2_{\eta,\gamma}(\tilde{A}) = \eta\ TS(\tilde{A}) + (1-\eta)\ I_\gamma(\tilde{A}) \quad (10)$$

$$I^3_{\eta,\gamma}(\tilde{A}) = \eta\ LS(\tilde{A}) + (1-\eta)\ I_\gamma(\tilde{A}) \quad (11)$$

$$I^4_{\eta,\gamma}(\tilde{A}) = \eta\ RS(\tilde{A}) + (1-\eta)\ I_\gamma(\tilde{A}) \quad (12)$$

where $\eta \in [0,1]$ is the index of modality which signifies the importance of central value or spreads.

3.2 Working Rule for ordering generalized LR fuzzy numbers

Consider two generalized LR fuzzy numbers $\tilde{A} = (m_1, n_1, \alpha_1, \beta_1; w_1)$. and $\tilde{B} = (m_2, n_2, \alpha_2, \beta_2; w_2)$..

Calculate centroid of centroids of $\tilde{A}$ *and* $\tilde{B}$ i.e., $G_{\tilde{A}}(x,y)$ and $G_{\tilde{B}}(x,y)$ by using Eq. (1) and $R_{\tilde{A}}$ and $R_{\tilde{B}}$ by using Eq. (3).

Then the following properties are used for ranking:

(i) If $R_{\tilde{A}} > R_{\tilde{B}}$ then $\tilde{A} > \tilde{B}$

(ii) If $R_{\tilde{A}} < R_{\tilde{B}}$ then $\tilde{A} < \tilde{B}$

(iii) If $R_{\tilde{A}} = R_{\tilde{B}}$ then the discrimination of fuzzy numbers is attained by using the index of optimism $\gamma \in [0,1]$and an index of modality $\eta \in [0,1]$ and the ranking is done in the following steps:

Step 1: Use Eq. (8) and find $I_\gamma(\tilde{A})$ and $I_\gamma(\tilde{B})$.

Step 2: Use Eq. (9) and find $I^1_{\eta,\gamma}(\tilde{A})$ and $I^1_{\eta,\gamma}(\tilde{B})$.

(i) If $I^1_{\eta,\gamma}(\tilde{A}) > I^1_{\eta,\gamma}(\tilde{B})$ *then* $\tilde{A} > \tilde{B}$

(ii) If $I^1_{\eta,\gamma}(\tilde{A}) < I^1_{\eta,\gamma}(\tilde{B})$ *then* $\tilde{A} < \tilde{B}$

(iii) If $I^1_{\eta,\gamma}(\tilde{A}) = I^1_{\eta,\gamma}(\tilde{B})$ *then* $\tilde{A} = \tilde{B}$, then go to step 3.

Step 3: Use Eq. (10) and find $I^2_{\eta,\gamma}(\tilde{A})$ and $I^2_{\eta,\gamma}(\tilde{B})$.

(i) If $I^2_{\eta,\gamma}(\tilde{A}) > I^2_{\eta,\gamma}(\tilde{B})$ *then* $\tilde{A} > \tilde{B}$

(ii) If $I^2_{\eta,\gamma}(\tilde{A}) < I^2_{\eta,\gamma}(\tilde{B})$ *then* $\tilde{A} < \tilde{B}$

(iii) If $I^2_{\eta,\gamma}(\tilde{A}) = I^2_{\eta,\gamma}(\tilde{B})$ *then* $\tilde{A} = \tilde{B}$, then go to step 4.

Step 4: Use Eq. (11) and find $I^{3}_{\eta,\gamma}\left(\tilde{A}\right)$ and $I^{3}_{\beta,\gamma}\left(\tilde{B}\right)$.

(i) If $I^{3}_{\beta,\gamma}\left(\tilde{A}\right) > I^{3}_{\beta,\gamma}\left(\tilde{B}\right)$ *then* $\tilde{A} > \tilde{B}$

(ii) If $I^{3}_{\eta,\gamma}\left(\tilde{A}\right) < I^{3}_{\eta,\gamma}\left(\tilde{B}\right)$ *then* $\tilde{A} < \tilde{B}$

(iii) If $I^{3}_{\eta,\gamma}\left(\tilde{A}\right) = I^{3}_{\eta,\gamma}\left(\tilde{B}\right)$ *then* $\tilde{A} = \tilde{B}$, then go to step 5.

Step 5: Use Eq. (12) and find $I^{4}_{\eta,\gamma}\left(\tilde{A}\right)$ and $I^{4}_{\eta,\gamma}\left(\tilde{B}\right)$.

(i) If $I^{4}_{\eta,\gamma}\left(\tilde{A}\right) > I^{4}_{\eta,\gamma}\left(\tilde{B}\right)$ *then* $\tilde{A} > \tilde{B}$

(ii) If $I^{4}_{\eta,\gamma}\left(\tilde{A}\right) < I^{4}_{\eta,\gamma}\left(\tilde{B}\right)$ *then* $\tilde{A} < \tilde{B}$

(iii) If $I^{4}_{\eta,\gamma}\left(\tilde{A}\right) = I^{4}_{\eta,\gamma}\left(\tilde{B}\right)$ *then* $\tilde{A} = \tilde{B}$, then go to step 6.

Step 6: Identify the values of w_1 and w_2

(i) If $w_1 > w_2$ *then* $\tilde{A} > \tilde{B}$

(ii) If $w_1 < w_2$ *then* $\tilde{A} < \tilde{B}$

(iii) If $w_1 = w_2$ *then* $\tilde{A} \approx \tilde{B}$

3.3 Numerical examples

In this section, some sets of fuzzy numbers are consider from the existing methods Chen and Sanguansat [14], Xu *et al.* [15], Cheng [16], Chu and Tsao [17], Yagger [18]. The given ranking method is explained by ranking these fuzzy numbers and represented diagrammatically as shown in the figure 2. The comparative study between the existing methods and proposed method is given in Table 1.

Set1: $\tilde{A} = (-0.1, 0.1, 0.1, 0.1; 0.4)$ $\tilde{B} = (0.0, 0.0, 0.1, 0.1; 0.4)$

Using Eq.(1)

$G_{\tilde{A}}(x, y) = (0, 0.15555)$, and $G_{\tilde{B}}(x, y) = (0, 0.15555)$

Using Eq. (3),

$R_{\tilde{A}} = 0.1555$ *and* $R_{\tilde{B}} = 0.1555$

$R_{\tilde{A}} = R_{\tilde{B}}$, so go to step1

Step1: $I_{\gamma}(\tilde{A}) = 0.07775$; $I_{\gamma}(\tilde{B}) = 0.07775$

Step2: $I^{1}_{\eta,\gamma}\left(\tilde{A}\right) = I^{1}_{\eta,\gamma}\left(\tilde{B}\right) = 0.0388$, *then* $R_{\tilde{A}} = R_{\tilde{B}}$, so goto step3

Step3: $I^{2}_{\eta,\gamma}\left(\tilde{A}\right) = 0.2388$ and $I^{2}_{\eta,\gamma}\left(\tilde{B}\right) = 0.1388$

$Since\ I^{2}_{\eta,\gamma}\left(\tilde{A}\right) > I^{2}_{\eta,\gamma}\left(\tilde{B}\right) \Rightarrow \tilde{A} > \tilde{B}$

By given method the ranking order is $\tilde{A} > \tilde{B}$. But according to Chen and Sanguansat [14], Xu *et al.* [15] and Yagger[18] the ranking order is $\tilde{A} = \tilde{B}$ and Cheng [16], Chu and Tsao [17] methods are not applicable to rank these fuzzy numbers. From figure 2 ,of set1, it is easy to see that the ranking results obtained by the existing methods are unreasonable and are not consistent with human instinct. Therefore given method overcame the drawbacks of the existing methods and the approach is simpler in calculations.

Set2: $\tilde{A} = (-0.2,0.3,0.3,0.1;0.6)$ $\tilde{B} = (-0.3,0.4,0.3,0.1;0.6)$

Using Eq.(1)

$G_{\tilde{A}}(x\ ,y) = (0.0277, 0.2333)$, and $G_{\tilde{B}}(x\ ,y) = (0.0277, 0.2333)$

Using Eq. (3),

$R_{\tilde{A}} = 0.2349\ \ and\ \ R_{\tilde{B}} = 0.2349$

$R_{\tilde{A}} = R_{\tilde{B}}$ so go to step1

Step1: $I_{\gamma}(\tilde{A}) = 0.1305\ ;\ I_{\gamma}(\tilde{B}) = 0.1305$

Step2: $I^{1}_{\eta,\gamma}\left(\tilde{A}\right) = I^{1}_{\eta,\gamma}\left(\tilde{B}\right) = 0.0902\ ,\ then\ R_{\tilde{A}} = R_{\tilde{B}}$, so goto step3

Step3: $I^{2}_{\eta,\gamma}\left(\tilde{A}\right) = 0.5152$ and $I^{2}_{\eta,\gamma}\left(\tilde{B}\right) = 0.6152$

$Since\ I^{2}_{\eta,\gamma}\left(\tilde{A}\right) < I^{2}_{\eta,\gamma}\left(\tilde{B}\right) \Rightarrow \tilde{A} < \tilde{B}$.

By given method the ranking order is $\tilde{A} < \tilde{B}$. But according to Chen and Sanguansat [14], Xu *et al.* [15] and Yagger[18] the ranking order is $\tilde{A} = \tilde{B}$ and Cheng [16], Chu and Tsao [17] methods are not applicable to rank these fuzzy numbers. From figure 2 ,of set2, it is easy to see that the ranking results obtained by the existing methods are unreasonable and are not consistent with human instinct. Therefore given method overcame the drawbacks of the existing methods and the approach is simpler in calculations.

Set3: $\tilde{A} = (-0.2,0.2,0.1,0.1;0.7)$ $\tilde{B} = (-0.1,-0.1,0,0.2;0.7)$

Using Eq.(1)

$G_{\tilde{A}}(x\ ,y) = (0,0.2722)$, and $G_{\tilde{B}}(x\ ,y) = (-0.0777,0.2722)$

Using Eq. (3),

$R_{\tilde{A}} = 0.2722 \text{ and } R_{\tilde{B}} = 0.28311$

$R_{\tilde{A}} < R_{\tilde{B}} \Rightarrow \tilde{A} < \tilde{B}$.

By given method the ranking order is $\tilde{A} < \tilde{B}$. But according to Chen and Sanguansat [14], Xu *et al.* [15] and Yagger[18] the ranking order is $\tilde{A} = \tilde{B}$ and Cheng [16], Chu and Tsao [17] methods are not applicable to rank these fuzzy numbers. From fiigure 2 ,of set3, it is easy to see that the ranking results obtained by the existing methods are unreasonable and are not consistent with human instinct. Therefore given method overcame the drawbacks of the existing methods and the approach is simpler in calculations.

Set4: $\tilde{A} = (0.0,0.1,0.4,0.2;0.8)$ $\tilde{B} = (0.0,0.1,0.3,0;0.8)$

Using Eq.(1)

$G_{\tilde{A}}(x\ ,y) = (0.0277,0.3111)$, and $G_{\tilde{B}}(x\ ,y) = (0.0166,0.3111)$

Using Eq. (3),

$\therefore R_{\tilde{A}} = 0.3123 \text{ and } R_{\tilde{B}} = 0.3115$

$R_{\tilde{A}} > R_{\tilde{B}} \Rightarrow \tilde{A} > \tilde{B}$.

By given method the ranking order is $\tilde{A} > \tilde{B}$. But according to Chen and Sanguansat [14], Xu *et al.* [15] and Yagger[18] the ranking order is $\tilde{A} = \tilde{B}$ and Cheng [16], Chu and Tsao [17] methods are not applicable to rank these fuzzy numbers. From figure 2 ,of set4, it is easy to see that the ranking results obtained by the existing methods are unreasonable and are not consistent with human instinct. Therefore given method overcame the drawbacks of the existing methods and the approach is simpler in calculations.

Set5: $\tilde{A} = (-0.3,-0.3,0.2,0.2;1)$ $\tilde{B} = (0,0,0,0;1)$ $\tilde{C} = (0,0,0,0;0.8)$

Using Eq.(1)

$G_{\tilde{A}}(x\ ,y) = (-0.3,0.3888)$, $G_{\tilde{B}}(x\ ,y) = (0,0.3888)$ and $G_{\tilde{C}}(x\ ,y) = (0,0.3111)$

Using Eq. (3),

$R_{\tilde{A}} = 0.4911,\ R_{\tilde{B}} = 0.3888, \text{ and } R_{\tilde{C}} = 0.3111$

$R_{\tilde{A}} > R_{\tilde{B}} > R_{\tilde{C}} \Rightarrow \tilde{A} > \tilde{B} > \tilde{C}$.

By given method the ranking order is $\tilde{A} > \tilde{B} > \tilde{C}$. But according to Chen and Sanguansat [14], Xu *et al.* [15] and Yagger[18] the ranking order is $\tilde{A} = \tilde{B} = \tilde{C}$ and Cheng [16], Chu and Tsao [17] methods are not applicable to rank these fuzzy numbers. From figure 2 ,of set5, it is easy to see that the ranking results obtained by the existing methods are unreasonable and are not consistent with human instinct. Therefore given method overcame the drawbacks of the existing methods and the approach is simpler in calculations.

Set6: $\tilde{A} = (0,0,0.2,0.2;0.8)$ $\tilde{B} = (0,0,0.2,0.2;1)$.

Using Eq.(1)

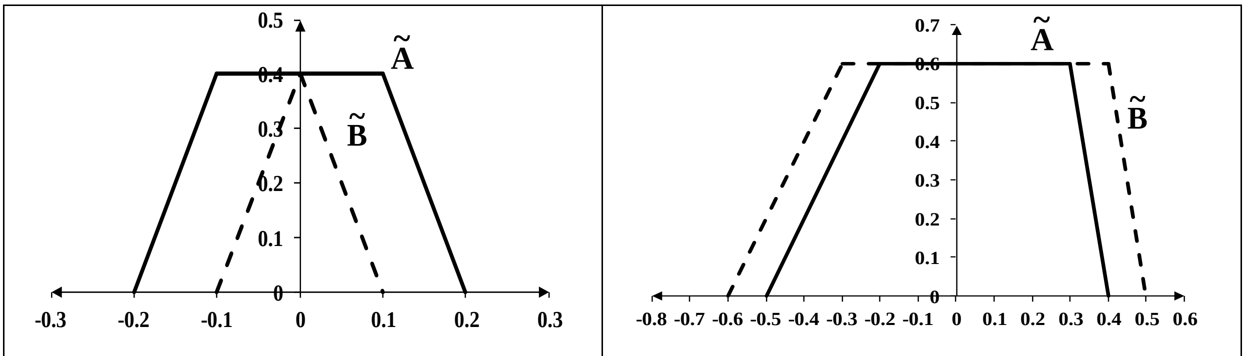
$G_{\tilde{A}}(x\ ,y) = (0,0.3111)$, and $G_{\tilde{B}}(x\ ,y) = (0, 0.3888)$

Using Eq. (3),

$R_{\tilde{A}} = 0.3111\ \ and\ \ R_{\tilde{B}} = 0.3888$

$R_{\tilde{A}} < R_{\tilde{B}} \Rightarrow \tilde{A} < \tilde{B}.$

By given method the ranking order is $\tilde{A} < \tilde{B}$. But according to Chen and Sanguansat [14], Xu *et al.* [15] and Yagger[18] the ranking order is $\tilde{A} = \tilde{B}$ and Cheng [16], Chu and Tsao [17] methods are not applicable to rank these fuzzy numbers. From figure 2 ,of set6, it is easy to see that the ranking results obtained by the existing methods are unreasonable and are not consistent with human instinct. Therefore given method overcame the drawbacks of the existing methods and the approach is simpler in calculations.



Set1: $\tilde{A} = (-0.1, 0.1, 0.1, 0.1; 0.4)$ $\tilde{B} = (0.0, 0.0, 0.1, 0.1; 0.4)$

Set2: $\tilde{A} = (-0.2, 0.3, 0.3, 0.1; 0.6)$ $\tilde{B} = (-0.3, 0.4, 0.3, 0.1; 0.6)$

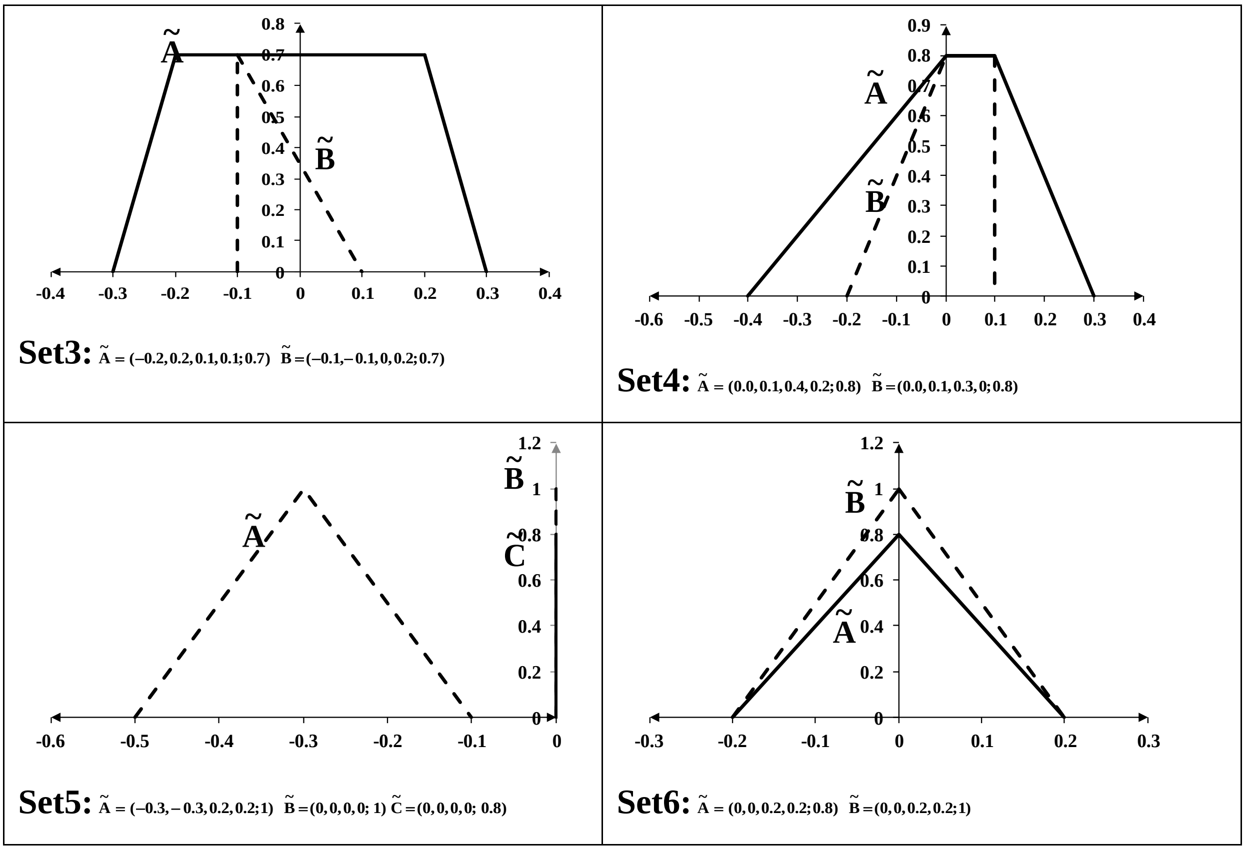


Figure 2. Sets of fuzzy numbers

Table 1. Comparison of existing methods and proposed method.

Methods	Set1	Set2	Set3	Set4	Set5	Set6
Chen and Sanguansat [15]	$\tilde{A}=\tilde{B}$	$\tilde{A}=\tilde{B}$	$\tilde{A}=\tilde{B}$	$\tilde{A}=\tilde{B}$	$\tilde{A}=\tilde{B}=\tilde{C}$	$\tilde{A}=\tilde{B}$
Xu *et al.* [22]	$\tilde{A}=\tilde{B}$	$\tilde{A}=\tilde{B}$	$\tilde{A}=\tilde{B}$	$\tilde{A}=\tilde{B}$	$\tilde{A}=\tilde{B}=\tilde{C}$	$\tilde{A}=\tilde{B}$
Cheng [23]	*	*	*	*	*	*
Chu and Tsao [7]	*	*	*	*	*	*
Yagger [4]	$\tilde{A}=\tilde{B}$	$\tilde{A}=\tilde{B}$	$\tilde{A}=\tilde{B}$	$\tilde{A}=\tilde{B}$	$\tilde{A}=\tilde{B}=\tilde{C}$	$\tilde{A}=\tilde{B}$
Proposed Method	$\tilde{A}<\tilde{B}$	$\tilde{A}<\tilde{B}$	$\tilde{A}>\tilde{B}$	$\tilde{A}>\tilde{B}$	$\tilde{B}>\tilde{C}>\tilde{A}$	$\tilde{A}<\tilde{B}$

Note: '*' denotes not applicable

4. Formulation of Fuzzy Linear Programming

In this Section, fuzzy linear programming problem is considered with fuzzy objective function coefficients, fuzzy constraints coefficients and fuzzy decision variables, mathematical formulation of fuzzy linear programming model, and working rule to find the fuzzy optimal value is presented.

4.1 Mathematical Formulation of Fuzzy Linear Programming Problem

Mathematically, the fuzzy linear programming problem can be stated as:

Maximize (or Minimize) $\tilde{Z} = \sum_{j=1}^{n} \tilde{c}_j \otimes x_j$

subject to

$$\sum_{j=1}^{n} \tilde{a}_{ij} \otimes x_j \leq,=,\geq \tilde{b}_i \,,\ i=1,2,...,m$$

$$x_j \geq 0, \forall\ j$$

where $\tilde{a}_{ij}$, $\tilde{b}_i$ and $\tilde{c}_j$ are *LR* generalized fuzzy numbers, and x_j are variables.

4.2 Working Rule to find the fuzzy optimal value

Step 1: Convert the considered problem into the following fuzzy linear programming problem:

Maximize (or Minimize) $\tilde{Z} = \sum_{j=1}^{n} \tilde{c}_j \otimes x_j$

subject to

$$\sum_{j=1}^{n} \tilde{a}_{ij} \otimes x_j \leq,=,\geq \tilde{b}_i \,,\ i=1,2,...,m$$

$$x_j \geq 0, \forall\ j$$

Step 2: Convert the fuzzy linear programming problem of step 1 into the following crisp Linear Programming Problem

Maximize (or Minimize) $\tilde{Z} = \sum_{j=1}^{n} R_{\tilde{c}_j} \otimes x_j$

subject to

$$\sum_{j=1}^{n} R_{\tilde{a}_{ij}} \otimes x_j \leq,=,\geq R_{\tilde{b}_i} \,,\; i=1,2,...,m$$
$$x_j \geq 0, \forall \; j$$

Step 3: Calculate the values of $R_{\tilde{c}_j}$, $R_{\tilde{a}_{ij}}$ *and* $R_{\tilde{b}_i}$ by using Eq.1 i.e., ranking function defined in section 3.1, and substitute the values in the following crisp Linear Programming Problem

Maximize (or Minimize) $\tilde{Z} = \sum_{j=1}^{n} R_{\tilde{c}_j} \otimes x_j$

subject to

$$\sum_{j=1}^{n} R_{\tilde{a}_{ij}} \otimes x_j \leq,=,\geq R_{\tilde{b}_i} \,,\; i=1,2,...,m$$
$$x_j \geq 0, \forall \; j$$

Step 4: For the crisp linear programming problem obtained in step 3, the optimal solution x_j is obtained by using the software applications like TORA or LINGO.

Step 5: The fuzzy optimal value of fuzzy linear programming problem is obtained by substituting the values of x_j in $\sum_{j=1}^{n} \tilde{c}_j \otimes x_j$

5. Numerical example

In this section a numerical example is considered to exemplify the fuzzy linear programming problem.

RMC, Inc., is a small firm that produces a variety of chemical-based products. In a particular production process, three raw materials are used to produce two products: a fuel additive and a solvent base. The fuel additive is sold to oil companies and is used in the production of gasoline and related fuels. The solvent base is sold to a variety of chemical firms and is used in both home and industrial cleaning products. The three raw materials are blended to form the fuel additive and solvent base as indicated in Table 2. It shows that a fuel additive is a mixture of material 1 and material 3. A solvent base is a mixture of material 1, material 2 and material 3. Due to the spoilage and the nature of the production process, any materials not used for current production are useless and may be discarded, to avoid these situations we considered the fuzzy linear programming and all the parameters i.e., material requirements, material available, and profit contribution are considered as LR fuzzy numbers.

Table 2. Material Requirements

Product		
	Fuel Additive	**Solvent Base**
Material 1	(0.4, 0.6, 0.2, 0.3; 1)	(0.5, 0.7, 0.2, 0.3; 1)
Material 2	-	(0.2, 0.3, 0.1, 0.2; 1)
Material 3	(0.6, 0.9, 0.3,0.4; 1)	(0.3, 0.5, 0.2, 0.3; 1)

RMC's production is constrained by a limited availability of the three raw materials. For the current production period, RMC has available the following fuzzy quantities of each raw material:

Material	**Amount Available for Production(tons)**
Material 1	(20, 26, 6, 2; 1)
Material 2	(5, 7, 3, 2; 1)
Material 3	(14, 20, 3, 4; 1)

The accounting department analyzed the production figures, assigned all relevant costs, and arrived at prices for both products that will result in a profit contribution of *($40, $48, $5, $2; 1)* for every ton of fuel additive produced and *($30, $36,$5, $6; 1)* for every ton of solvent base produced. Determine the number of tons of fuel additive and the number of tons of solvent base to produce in ordered to maximize the fuzzy total profit contribution .

The fuzzy optimal value of the chosen fuzzy linear programming can be obtained by using the following steps

Step 1: As mentioned in section 4.1, the given problem is converted as

$$\text{Maximize: } ((40, 48, 5, 2; 1) \otimes x_1) \oplus ((30, 36, 5, 6; 1) \otimes x_2))$$

subject to

$$((0.4, 0.6, 0.2, 0.3; 1) \otimes x_1)) \oplus ((0.5, 0.7, 0.2, 0.3; 1) \otimes x_2)) \leq (20, 26, 6, 2; 1)$$

$$((0.2, 0.3, 0.1, 0.2; 1) \otimes x_2)) \leq (5, 7, 3, 2; 1)$$

$$((0.6, 0.9, 0.3, 0.4; 1) \otimes x_1)) \oplus ((0.3, 0.5, 0.2, 0.3; 1) \otimes x_2)) \leq (14, 20, 3, 4; 1)$$

$$x_1, x_2 \geq 0$$

Step 2: By using step 2 of section 4.2 gives the following crisp Linear Programming Problem

$$\text{Maximize: } (R(40, 48, 5, 2; 1) \otimes x_1) \oplus (R(30, 36, 5, 6; 1) \otimes x_2))$$

subject to

$$(R(0.4, 0.6, 0.2, 0.3; 1) \otimes x_1)) \oplus (R(0.5, 0.7, 0.2, 0.3; 1) \otimes x_2)) \leq R(20, 26, 6, 2; 1)$$

$$(R(0.2, 0.3, 0.1, 0.2; 1) \otimes x_2)) \leq R(5, 7, 3, 2; 1)$$

$$(R(0.6, 0.9, 0.3, 0.4; 1) \otimes x_1)) \oplus (R(0.3, 0.5, 0.2, 0.3; 1) \otimes x_2)) \leq R(14, 20, 3, 4; 1)$$

$$x_1, x_2 \geq 0.$$

Step 3: By using step 3 of section 4.2, it is converted as

$$\text{Maximize: } (43.66) \otimes x_1 \oplus (33.11) \otimes x_2)$$

subject to

$$(0.642) \otimes x_1 \oplus (0.724) \otimes x_2 \leq 22.55$$

$$(0.468) \otimes x_2 \leq 5.901$$

$$(0.854) \otimes x_1 \oplus (0.565) \otimes x_2 \leq 17.11$$

$$x_1, x_2 \geq 0.$$

Step 4: By using step 4 of section 4.2, the optimal solution is:

$$x_1 = 11.68 \text{ and } x_2 = 12.83$$

Step 5: By using step 5 of section 4.2, the maximize fuzzy total profit contribution is:

(852.1, 1022.52, 122.55, 100.34).

6. Conclusion

The given ranking method can rank generalized *LR* fuzzy numbers, their crisp numbers and images which are special case of fuzzy numbers, and can discriminate fuzzy ranking in case of equal ranks. This method gives satisfactory results and correct ranking order to well defined problems which were failed by most of the existing methods like, Chen and Sanguansat [14], Xu et al. [15], Cheng [16], Chu and Taso [17] and Yager [18]. The given method is easy to apply when compared with some of the existing methods and reveals that they are more reliable in the point of view of optimality. This ranking method is applied to the proposed fuzzy linear programming problem to find the fuzzy optimal value. This new ranking method is a powerful method to deal with fuzzy linear programming problem as it considers the decision maker's view in both the stages i.e., ranking of fuzzy numbers and finding the fuzzy optimal value. And it can be applied to various real world problems.

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