Various Graph Labeling Techniques on H-Graph

Srushti Gandhi[1], Dharamvirsinh Parmar[2*]

[1]*M.Phil. Scholar at Department of Mathematics, C.U. Shah University*, Wadhwan, India
[2]*Assistant Professor at Department of Mathematics, C.U.Shah University*, Wadhwan, India
[1]gandhi.srushti@gmail.com, [2]dharamvir_21@yahoo.co.in

Abstract

The H-Graph of path P_n is the graph obtained from two copies of path P_n with the vertices $v_1, v_2, v_3, v_4, \ldots, v_n$ and $u_1, u_2, u_3, u_4, \ldots, u_n$ by joining the vertices $v_{\frac{n+1}{2}}$ and $u_{\frac{n+1}{2}}$ if n is odd and $v_{\frac{n}{2}+1}$ and $u_{\frac{n}{2}}$ if n is even. In this paper we discuss square sum labeling, mean labeling, prime labeling, sum labelling and sum perfect square labeling of H-Graph.

Keywords: *H-graph, Mean labeling, Prime labeling, Square sum labeling, Sum perfect square labeling.*

1. INTRODUCTION

We consider a finite, connected and undirected graph without loops. Graph $G = (V, E)$ having set ofvertices $V(G)$ and set of edges $E(G)$ respectively. Labeling of a graph G is an assignment of integers either to the vertices or edges or both subject to certain conditions. For detailed survey on graph labeling we refer to the dynamic survey of graph labeling by J.A.Gallian.

Definition:1.1 The H-graph of path $\mathrm{P_n}$ is the graph obtained from two copies of $\mathrm{P_n}$ with the vertices $\mathrm{v_1, v_2, v_3, v_4, \ldots, v_n}$ and $\mathrm{u_1, u_2, u_3, u_4, \ldots, u_n}$ by joining the vertices $\mathrm{v}_{\frac{n+1}{2}}$ and $\mathrm{u}_{\frac{n+1}{2}}$ if n is odd and $\mathrm{v}_{\frac{n}{2}+1}$ and $\mathrm{u}_{\frac{n}{2}}$ if n is even.

Definition:1.2 Let $G = (V, E)$ be a graph with p vertices and q edges. A bijection map $f: V \to \{0,1,2,3, \ldots, p-1\}$ is called sum perfect square labeling of G, if the induced edge function $\mathrm{f}^*: E \to \mathbb{N}$ defined by $\mathrm{f}^*(uv) = [\mathrm{f(u)} + \mathrm{f(v)}]^2$ for all edges uv is injective. A graph which admits sum perfect square labeling is called sum perfect square graph.

Definition 1.3 Let $G = (V, E)$ be a graph with p vertices and q edges. A injective map $f: V \to \{0,1,2,3, \ldots, q\}$ is called mean labeling if the induced edge function $\mathrm{f}^*: E \to \mathbb{N}$ defined by

$$\mathrm{f}^*(\mathrm{e = uv}) = \frac{\mathrm{f(u)+f(v)}}{2}; \qquad \text{if } f(u) + f(v) \text{ is even}$$

$$\mathrm{f}^*(\mathrm{e = uv}) = \frac{\mathrm{f(u)+f(v)+1}}{2}; \qquad \text{if } f(u) + f(v) \text{ is odd}$$

*Corresponding Author

A graph that admits mean labeling is called mean labeling graph.

Definition:1.4 Let $G = (V, E)$ be a graph with p vertices and q edges. An injective map $f: V \to \{1,2,3, \dots, |V|\}$ is called prime labeling of graph G if for every pair of adjacent vertices u and v, $gcd(f(u), f(v)) = 1$. A graph which admits prime labeling is called prime labeling graph.

Definition:1.5 Let $G = (V, E)$ be a graph with p vertices and q edges. An bijective map $f: V \to \{0,1,2,3, \dots, p-1\}$ is said to be square sum labeling of graph G, if the induced edge function $f^*: E \to \mathbb{N}$ defined by $f^*(uv) = f(u)^2 + f(v)^2$, for every uv ϵ E(G) is injective. The graph that admits prime labeling is called prime labeling graph.

2. Main Results

Theorem 2.1: H-graph is sum perfect square labeling.

Proof: Let G be a H-graph; $\{u_i, v_i / 1 \le i \le n,\}$ be the set of vertices and $p = |V| = 2n$ and $q = |E| = 2n - 1$.

Define: $f: V \to \{0,1,2,3, \dots, 2n-1\}$ by:

For $1 \le i \le n$;

$f(u_i) = i - 1$

$f(v_i) = n + i - 1$

Which is bijective function.

Then the induced edge labels are:

$f^*(u_i u_{i+1}) = (2i-1)^2$ $\quad 1 \le i \le n$

$f^*(v_i v_{i+1}) = (2n + 2i - 1)^2$; $\quad 1 \le i \le n$

$f^*(u_{\frac{n+1}{2}} v_{\frac{n+1}{2}}) = (2n-1)^2$; $\quad$ if n is odd

$f^*(u_{\frac{n}{2}+1} v_{\frac{n}{2}}) = (2n-1)^2$; $\quad$ if n is even.

Which is injective function. Hence H-graph is perfect square sum labeling.

Example 2.2 Square sum labeling of H-graph for $n = 3$ and $n = 4$.

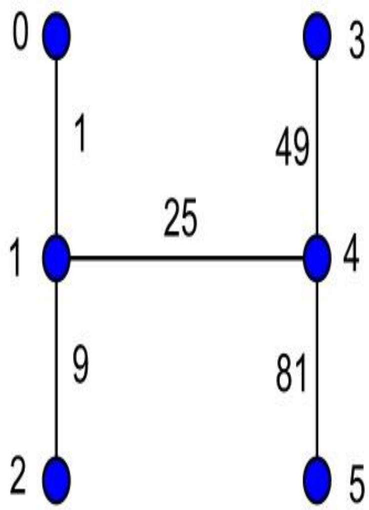

Figure 1(a) for $n = 3$

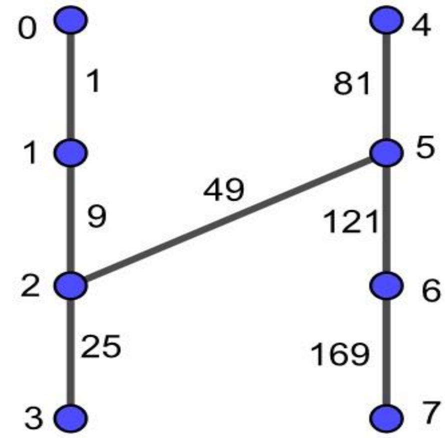

Figure 1(b) for $n = 4$.

Figure 1. Perfect Square Sum labeling of H – graph with $n = 3$ and $n = 4$ vertices

Theorem 2.3: H-graph is mean labeling.

Proof: Let G be a H-graph; $\{u_i, v_i / 1 \leq i \leq n,\}$ be the set of vertices and $p = |V| = 2n$ and $q = |E| = 2n - 1$.

Define a function $f: V \rightarrow \{0,1,2,3, \dots ,2n - 1\}$ by

For $1 \leq i \leq n$;

$f(u_i) = i$-1

$f(v_i) = n + i - 1$

which is injective function.

Then the induced edge labels are:

$f^*(u_i u_{i+1}) = i$ $\quad 1 \leq i < n$

$f^*(v_i v_{i+1}) = n + i;$ $\quad 1 \leq i < n$

$f^*(u_{\frac{n+1}{2}} v_{\frac{n+1}{2}}) = n;$ $\quad$ if n is odd

$f^*(u_{\frac{n}{2}+1} v_{\frac{n}{2}}) = n;$ $\quad$ if n is even.

Which is injective function. Hence H-graph is mean labeling.

Example2.4: Mean labeling of H-graph for $n=5$ and $n=6$.

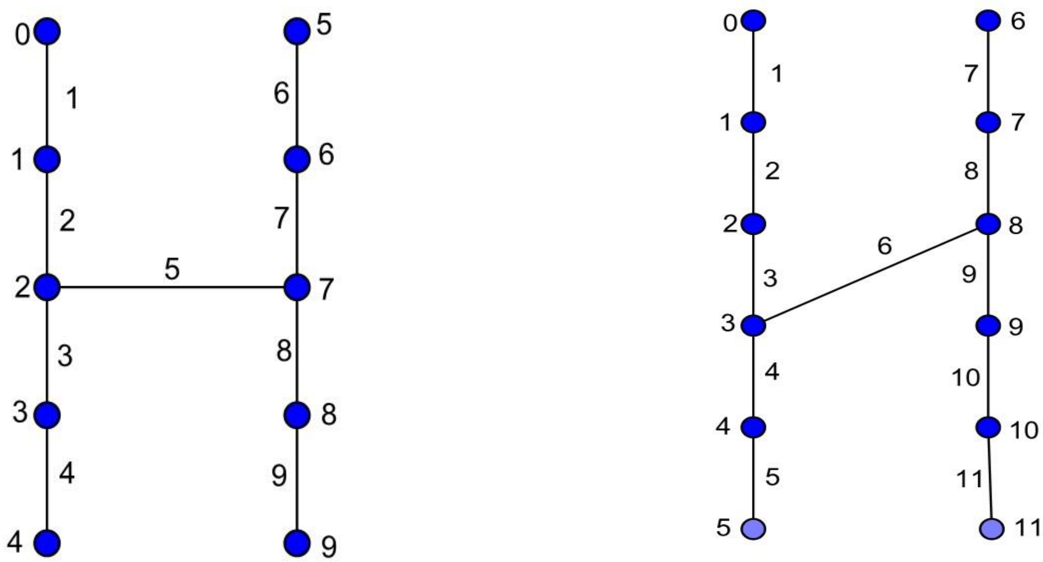

Figure 2(a): For $n=5$. **Figure 2(b): For $n=6$.**

Figure 2. Mean labeling of H – graph with $n = 5$ and $n = 6$ vertices

Theorem2.5: H-graph is prime labeling.

Proof: Let G be a H-graph; $\{u_i, v_i / 1 \leq i \leq n,\}$ be the set of vertices and $p = |V| = 2n$.

Define a function $f: V \rightarrow \{1,2,3, \dots, 2n\}$ by

Case:1 If $n \in \mathbb{N} - \{4^i / i \in \mathbb{N}\}$

$f(u_i) = i;$ $\quad 1 \leq i \leq n$

$f(v_i) = n + i$ $\quad 1 \leq i \leq n$

Which is injective function.

Case:2 If $n \in \{4^i / i \in \mathbb{N}\}$

$f(u_i) = i$ $\quad 1 \leq i \leq n$

$f(v_i) = n + i + 1$ $\quad 1 \leq i \leq n$

$f(v_{\frac{n}{2}}) = n + 1;$ $\quad 1 \leq i \leq n; i \neq \frac{n}{2}$

Which is injective function.

Hence, H-graph is prime labeling.

Example 2.6: Prime labeling of H-graph for $n = 9$ and $n = 16$.

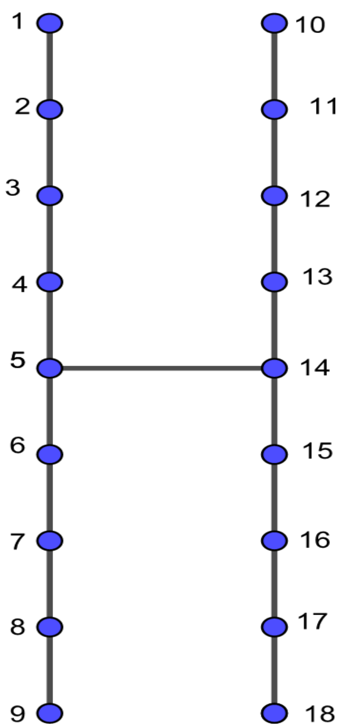

Figure 3(a): For $n = 9$.

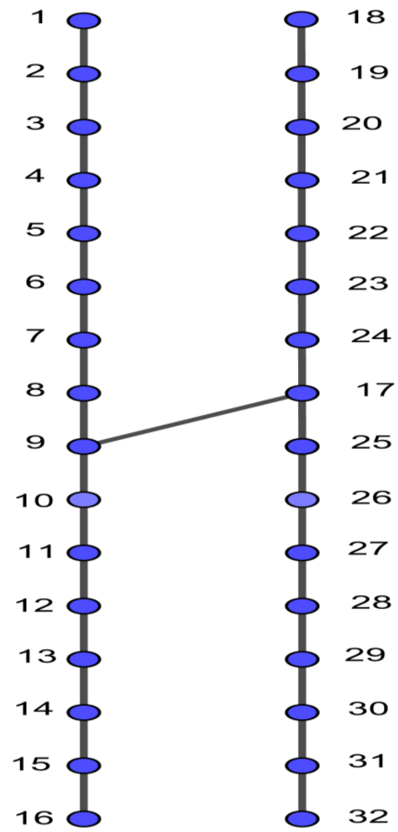

Figure 3(b): For $n = 16$.

Figure 3. Prime labeling of H – graph with $n = 9$ and $n = 16$ vertices

Theorem2.7: H-graph is square sum labeling.

Proof: Let G be a H-graph; $\{u_i, v_i / 1 \leq i \leq n,\}$ be the set of vertices and $p = |V| = 2n$ and $q = |E| = 2n - 1$.

Define a function $f: V \rightarrow \{0,1,2,3, \ldots, 2n-1\}$ by

For $1 \leq i \leq n$; ;

$f(u_i) = (i - 1)$

$f(v_i) = 2i - 1$

Then the induced edge labels are:

$f^*(u_i u_{i+1}) = 8i^2 - 8i + 4$ $\quad 1 \leq i \leq n$

$f^*(v_i v_{i+1}) = 8i^2 + 2$ $\quad 1 \leq i \leq n$

$f^*(u_{\frac{n+1}{2}} v_{\frac{n+1}{2}}) = (2n^2 - 2n + 1)$; if n is odd

$f^*(u_{\frac{n}{2}+1} v_{\frac{n}{2}}) = (2n^2 - 2n + 1)$; if n is even.

Which is injective.

Hence, H-graph is square sum labeling.

Example2.8: Square sum labeling of H-graph for $n = 8$ and $n = 11$.

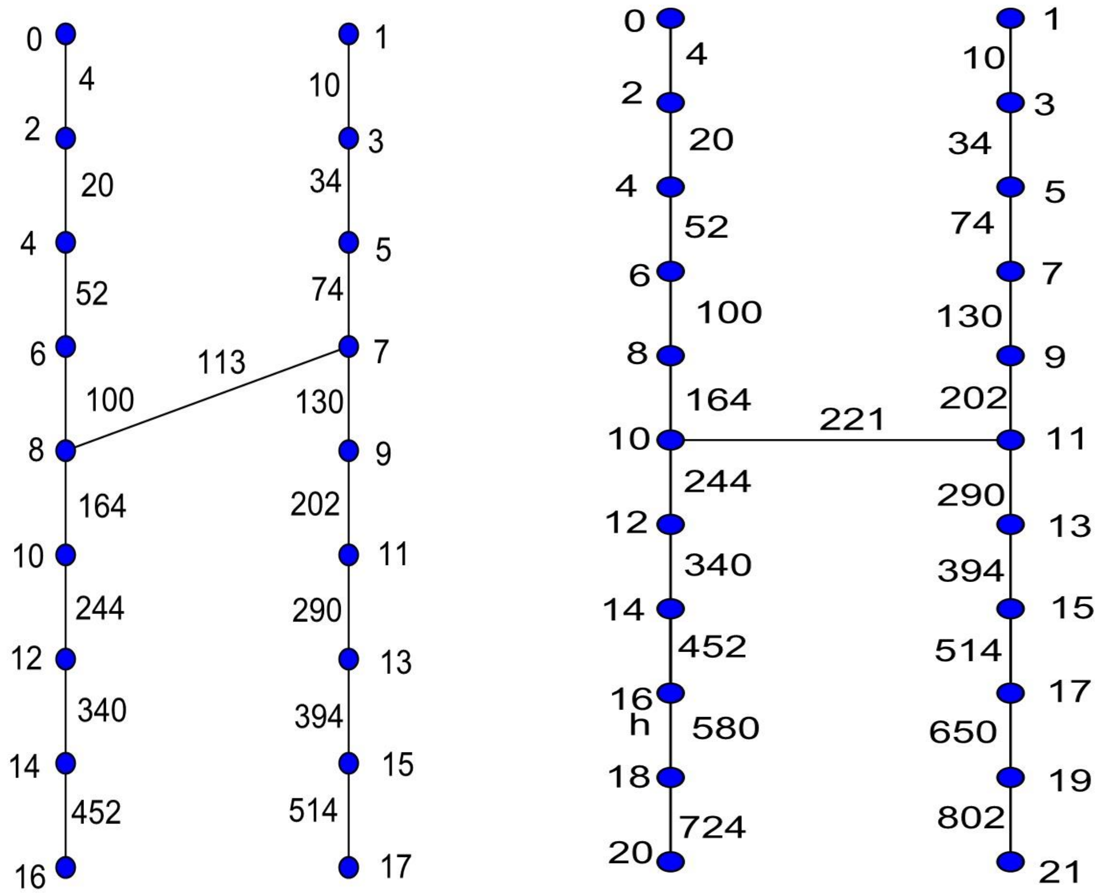

Figure 4(a) For $n = 8$ **Figure 4(b) For $n = 11$.**

Figure 4. Mean labeling of H – graph with $n = 8$ and $n = 11$ vertices

3. Conclusion

In this paper we have proved that H-graph are square sum labeling, sum perfect square labeling, prime labeling and mean labeling.

References

[1] A. Edward Samuel, S. Kalaivani, "Square sum labeling for some lilly related graphs", International Journal of Advanced Technology and Engineering Exploration, Vol 4, (2017), pp 68 – 72.

[2] D.Parmar, J. Joshi, "H_K - Cordial Labeling of Triangular Snake Graph", Journal of Applied Science and Computations. Vol VI, no III, (2019), pp 2118 – 2123.

[3] D.Parmar, U Vaghela, "Skolem Difference Mean Labeling of Some Path Related Graphs", Pramana Research Journal, Vol 9, no 5, (2019), pp 651-657.

[4] J A Gallian , "A dynamic survey of graph labeling" , The electronic Journal of Combinatories.,17(2017) #DS6

[5] J.Gross and J.Yellen, "Graph theory and its application", CRC Press,(1999).

[6] N.Revathi, "Mean labeling of some graphs", International journal of science and research, Vol 4 no. 10, (2015), pp-1921-1923.

[7] P.Shah, D.Parmar, "Integer Cordial Labeling of Triangular Snake Graph", International Journal of Scientific Research and Reviews Vol.8, no. 1, (2019), pp 3118-3126.

[8] S.Sakthivel, T.R.Karpagam, "Prime Labelling Of Some New Standard Graphs", Paripex - Indian Journal Of Research, Vol 6, no. 2, (2017),pp343 – 345.

[9] S.Sonchhatra. G.V.Ghodasara, "Some new Sum perfect square labeling of graphs", International journal of scientific and innovative mathematical research, Vol 4, no. 1, (2016), pp-64-70.