Regularity Concept of L-Fuzzy Bi-Ideals in Near-Ring

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Abstract:

In this paper, we apply the idea of near ring. We introduce the notion of L-fuzzy bi-ideal in near-ring. Also we give conditions for a near ring with unity to be strongly regular in terms of fuzzy set.

Keywords: *Near-ring, bi-ideals, fuzzy bi-ideals, L-fuzzy bi-ideals.*

1. Introduction

The fundamental concept of fuzzy set was introduced by Zadeh [10].Garrett Birchof [2] introduced the concept of lattice theory. Further J.A Goguen [3] replaced the valuations set [0,1] by means of a complete lattice in an attempt to make a generalized study of fuzzy set theory by studying L-fuzzy sets. Abouzaid [1] introduced the concept of fuzzy sub near-ring and studied fuzzy ideals in near-ring. Mason [4] introduced the notion of strong regularity of a near-ring. T.Manikantan [5] introduced the notion of a fuzzy bi-ideal of a near-ring. S.Narmada and V.Mahesh Kumar [6] introduced the notionof bi-ideals in near-ring. In this paper we study the notion of L-fuzzy bi-ideal in near-ring. We give characterizations of L-fuzzy bi-ideals in near-rings. We give conditions for a near-ring with unity to be strongly regular in terms of fuzzy set.

2. Preliminaries

For the sake of continuity we recall some basic definitions.

2.1 Definition:

A non empty set N with two binary operations '+' and '.' is called a near-ring if

$(i)\ (N,+)\ is\ a\ group.$

$(ii)\ (N,.)\ is\ a\ semigroup$

$(iii)\ x.(y+z)=x.y+x.z\ \ for\ all\ x,y,z\in N$

Example:

Let $N=\{p,q,r,s\}$ be a set with two binary operations as follows:

Table 1.

+	p	q	r	s
p	p	q	r	s
q	q	p	s	r
r	r	s	q	p
s	s	r	p	q

Table 2.

.	p	q	r	s
p	p	p	p	P
q	p	p	p	P
r	p	p	p	P
s	p	p	q	q

$(N,+,.)$ is a near-ring.

2.2 Definition:

A fuzzy set μ_A in a near ring N is called a fuzzy ideal of N if it satisfies:

(i) $\mu_A(x-y) \geq \min\{\mu_A(x), \mu_A(y)\}$

(ii) $\mu_A(y+x-y) \geq \mu_A(x)$

(iii) $\mu_A(xy) \geq \mu_A(y)$

$(iv) \mu_A((x+z)y-xy) \geq \mu_A(z)$ $for\, all\, x, y, z \in N$

2.3 Definition:

A fuzzy set μ_A in N is a fuzzy bi ideal of N if

(i) $\mu_A(x-y) \geq \min\{\mu_A(x), \mu_A(y)\}$ $for\, all\, x, y \in N$

(ii) $\mu_A(x\,y\,z) \geq \min\{\mu_A(x), \mu_A(z)\}$ $for\, all\, x, y, z \in N$

2.4 Definition:

A fuzzy set μ_A in N is a fuzzysub near-ring of N if,

(i) $\mu_A(x-y) \geq \min\{\mu_A(x), \mu_A(y)\}$

(ii) $\mu_A(x\,y) \geq \min\{\mu_A(x), \mu_A(y)\}$ $for\, all\, x, y, \in N$

2.5 Definition:

A fuzzy set μ_A in a set N is a function $\mu_A : N \to [0,1]$.Denote by $\mathrm{Im}(\mu_A)$ the image set of μ_A. For $t \in [0,1]$ the set $\mu_t^{\geq} = \{x \in N / \mu_A(x) \geq t\}$ (rep. $\mu_t^{\leq} = \{x \in N / \mu_A(x) \leq t\}$) is called a upper (resp.lower) t-level cut of μ_A.

3. MAIN RESULTS

3.1 Definition:

A fuzzy set μ_A in N is a L-fuzzy sub near-ring of N if for all $x, y \in N$,

(i) $\mu_A(x-y) \geq \mu_A(x) \wedge \mu_A(y)$

(ii) $\mu_A(x\,y) \geq \mu_A(x) \wedge \mu_A(y)$

3.2 Definition:

A fuzzy set μ_A in N is a L-fuzzy bi-ideal of N if for all $x, y, z \in N$,

(i) $\mu_A(x-y) \geq \mu_A(x) \wedge \mu_A(y)$

(ii) $\mu_A(x\,y\,z) \geq \mu_A(x) \wedge \mu_A(z)$

Example:

Consider the fuzzy subset μ_A of R defined by,

$$\mu_A(x) = \begin{cases} 0.6 & if \;\; x \in \langle 2 \rangle \\ 0.2 & otherwise \end{cases}$$

Then μ_A is L-fuzzy bi-ideal of N.

3.3 Theorem:

A fuzzy set μ_A in N is an L-fuzzy bi-ideal of N iff $\mu_A{}^C$ is an L-fuzzy bi-ideal of N.

Proof:

If μ_A is an L-fuzzy bi-ideal of N.

To prove:

$\mu_A{}^C$ is an L-fuzzy bi-ideal of N. $\mu_A{}^C$ is a L-fuzzy bi-ideal of N.

$$\mu_A{}^C(x-y)=1-\mu_A(x-y)$$
$$\geq 1-(\mu_A(x)\wedge\mu_A(y))$$
$$\geq (1-\mu_A(x))\wedge(1-\mu_A(y))$$
$$=\mu_A{}^C(x)\wedge\mu_A{}^C(y)$$
$$Therefore, \mu_A{}^C(x-y)\geq\mu_A(x)\wedge\mu_A{}^C(y)\ for\,all\,x,y\in N$$
$$\mu_A{}^C(x\,y\,z)=1-\mu_A(x\,y\,z)$$
$$\geq 1-(\mu_A(x)\wedge\mu_A(z))$$
$$\geq (1-\mu_A(x))\wedge(1-\mu_A(z))$$
$$=\mu_A{}^C(x)\wedge\mu_A{}^C(z)$$
$$Therefore, \mu_A{}^C(x\,y\,z)\geq\mu_A(x)\wedge\mu_A{}^C(z)\ for\,all\,x,y,z\in N$$

$\mu_A{}^C$ is an L-fuzzy bi-ideal of N.

Conversely, $\mu_A{}^C$ is an L-fuzzy bi-ideal of N, then clearly the condition (i) and (ii) of Definition 3.2 are valid.

3.4 Theorem:

A fuzzy set μ_A in N is a L-fuzzy bi-ideal of N iff all the non-empty sets $\mu_t^{\geq}$ and $\mu_t^{\leq}$ are bi-ideal ofN for all $t\in \mathrm{Im}(\mu_A)$

Proof:

Suppose that μ_A is an L-fuzzy bi-ideal of N.

For $x,y\in\mu_t^{\geq}$,we have $\mu_A(x-y)\geq\mu_A(x)\wedge\mu_A(y)$
$\geq t$

Therefore, $x-y\in\mu_t^{\geq}$

Let $x,z\in\mu_t^{\geq}$ and $y\in N$ Then $\mu_A(xyz)\geq\mu_A(x)\wedge\mu_A(z)$
$\geq t$

And so $xyz\in\mu_t^{\geq}$

Hence $\mu_t^{\geq}$ is aL-fuzzy bi-ideal of N for all $t\in\mathrm{Im}(\mu_A)$ also $\mu_t^{\leq}$ are L-fuzzy bi-ideal of N for all $t\in\mathrm{Im}(\mu_A)$

Conversely,

Suppose that $\mu_t^{\geq}$ and $\mu_t^{\leq}$ are bi-ideals of N for all $t\in\mathrm{Im}(\mu_A)$

Suppose that $x,y\in N$

$\mu_A(x-y)\leq\mu_A(x)\wedge\mu_A(y)$

Choose r, such that $\mu_A(x-y)<r<\mu_A(x)\wedge\mu_A(y)$ Then we get $x,y\in\mu_t^{\geq}$ but $x-y\notin\mu_t^{\geq}$ a contradiction, Therefore $\mu_A(x-y)\geq\mu_A(x)\wedge\mu_A(y)$

Similarly $\mu_A(x\,y\,z)\geq\ \mu_A(x)\wedge\mu_A(z)\ for\,all\,x,y,z\in N$

Hence μ_A is L-fuzzy bi-ideal of N.

3.5 Theorem:

A non empty set B of N is a bi-ideal of N iff $A=(\chi_B,\chi_B{}^C)$ is an L-fuzzy bi-ideal of N.

Proof:

Straightforward

4. L-FUZZY BI-IDEALS AND REGULARITY

A near-ring N is regular if for every $a \in N$ there is an $x \in N$ such that $a = axa$ .A near ring N is strongly regular if for every $a \in N$ there is an $x \in N$ such that $a = xa^2$

4.1 Theorem:

Every L-fuzzy bi-ideal in a regular near ring is a L-fuzzy sub near- ring of N.

Proof:

Let μ_A be a L-fuzzy bi-ideal of N and Let $a, b \in N$.

Since N is regular, there exist $x \in N$ such that $a = a\,x\,a$

$$Then \quad \mu_A(ab) = \mu_A((a\,x\,a)b)$$
$$= \mu_A(a(\,x\,a\,)b)$$
$$\geq \mu_A(a) \wedge \mu_A(b)$$
$$\therefore \mu_A(ab) \geq \mu_A(a) \wedge \mu_A(b)$$

Thus μ_A is a L-fuzzy sub near-ring of N.

4.2 Lemma:

Let N be a strongly regular near ring. If $a = xa^2$ for some $a, x \in N$, then $a = a\,x\,a$ and $a\,x = x\,a$

4.3 Theorem:

If N is strongly regular, then for each $a \in N$ there is some $y \in N$ such that $a = a^2ya^2$

Proof:

Since N is strongly regular for each $a \in N$ there is an $x \in N$ such that $a = xa^2$ then by lemma 4.2

$$a = a\,x\,a \quad and \quad a\,x = x\,a$$
$$\therefore a = a\,(a\,x) = a^2x \ \ and \ hence,$$
$$a = a\,x\,a = (a^2\,x)\,x\,(\,x\,a^2)$$
$$= a^2ya^2$$

4.4 Theorem:

Let N be a strongly regular near- ring.then for every L-fuzzy B-ideal in N, we have $\mu_A(x) = \mu_A(x^2)\ for\,all\,x \in N$.

Proof:

Let μ_A be a L-fuzzy Bi-ideal of N and let $x \in N$ since N is strongly regular, there exist $y \in N$ such that $x = x^2yx^2$ then,

$$\mu_A(x) = \mu_A(x^2y\,x^2)$$
$$\geq \mu_A(x^2) \wedge \mu_A(x^2)$$
$$= \mu_A(x^2)$$

4.5 Theorem:

Let N be a near- ring with identity1 if every L-fuzzy Bi-ideal μ_A in N satisfies $\mu_A(x) = \mu_A(x^2)\ for\,all\,x \in N$ then N is strongly regular.

Proof:

Suppose that every L-fuzzy Bi-ideal μ_A of N satisfies

$\mu_A(x) = \mu_A(x^2)\ for\,all\,x \in N$

Let $a \in N$ then $B = Na^2$ is a Bi-ideal o

$\therefore A = (\chi_B, \chi_B{}^C)$ is an L-fuzzy Bi-ideal of N ($\because$ by Theorem 3.5)

Since $a^2 \in B = Na^2, \chi_B(a^2) = 1.$

But by hypothesis $\chi_B(a) = \chi_B(a^2)$

$\therefore \chi_B(a) = 1$ and so $a \in B = Na^2$

Thus $a = xa^2$ $for\, some\, x \in N$

Hence N is strongly regular.

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