TRANSIENT THERMOELASTIC PROBLEMS OF SEMI-INFINITE HOLLOW CYLINDER ON OUTER CURVED SURFACE:DIRECT PROBLEM

Pallavi Meshram and N. W. Khobragade
Department of Mathematics, RTM Nagpur University, Nagpur (M.S), India.
E-mail: khobragadenw@gmail.com

Abstract

This paper is concerned with transient thermoelastic problem in which we need to determine the temperature distribution, displacement function and thermal stresses of a semi-infinite hollow cylinder when the boundary conditions are known. Integral transform techniques are used to obtain the solution of the problem. The expressions are obtained in terms of Bessel's function in the form of infinite series.

Key Words: Semi-infinite hollow cylinder, transient problem, Integral transform, direct problem

1. Introduction

In **1962**, **Nowacki [1]** has published a book on thermoelasticity**.** Sierakowski and Sun studied the direct problems of an exact solution to the elastic deformation of finite length hollow cylinder. **In 2009, Kamdi, Khobragade** and **Durge [3]** studied transient thermoelastic problem for a circular solid cylinder with radiation. **Walde** and **Khobragade [4**] discussed transient thermoelastic problem of a finite length hollow cylinder. **Kulkarni and Khobragade [5]** derived thermal stresses of a finite length hollow cylinder. **Warbhe** and **Khobragade [6]** discussed numerical study of transient thermoelastic problem of a finite length hollow cylinder. **Lamba** and **Khobragade [7]** studied analysis of coupled thermal stresses in a axisymmetric hollow cylinder**. Hiranwar** and **Khobragade [8**] studied thermoelastic problem of a cylinder with internal heat sources. **Bagde** and **Khobragade [9**] discussed **heat** conduction problem for a finite elliptic cylinder**.**

Khobragade, Khalsa and Kulkarni [10] investigated thermal deflection of a finite length hollow cylinder due to heat generation. **Khobragade [11]** studied **thermoelastic** analysis of a thick hollow cylinder with radiation conditions**. Ghume, Mahakalkar and Khobragade [12]** derived interior thermo elastic solution of a hollow cylinder. **Chauthale, Singru and Khobragade [13]** studied thermal stress analysis of a thick hollow cylinder. **Singru and Khobragade [14]** developed integral transform methods for inverse problem of heat conduction with known boundary of semi-infinite hollow cylinder and its stresses. **Fule, Warbhe and Khobragade [15]** derived thermal stresses of semi-infinite hollow cylinder with internal heat source. **Ozisik [16]** published a book on Boundary Value Problem of Heat Conduction.
In this paper, an attempt has been made to solve two inverse problems of thermoelasticity.

In both the problems, an attempt has been made to determine the temperature distribution. unknown temperature gradient, displacement function and thermal stresses on outer curved surface of semi-infinite hollow cylinder.

2. Statement of the Problem-I

Consider semi-infinite hollow cylinder occupying the space D : $a \le r \le b$, $0 \le z \le \infty$. The thermoelastic displacement function as [1] is governed by the Poisson's equation

$$\nabla^2\phi = \left(\frac{1+\nu}{1-\nu}\right)a_t T \tag{2.1}$$

with $\phi = 0$ at $r = a$ and $r = b$ (2.2) where $\nabla^2 = \frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r} + \frac{\partial^2}{\partial z^2}$

ν and a_t are the Poisson's ratio and the linear coefficient of thermal expansion of the material of the cylinder and T is the temperature of the cylinder satisfying the differential equation

$$\frac{\partial^2 T}{\partial r^2} + \frac{1}{r}\frac{\partial T}{\partial r} + \frac{\partial^2 T}{\partial z^2} = \frac{1}{k}\frac{\partial T}{\partial t} \quad (2.3)$$

subject to the initial conditions

$$T(r, z, 0) = 0 \quad (2.4)$$

the boundary conditions

$$T(a, z, t) = 0 \quad (2.5)$$

$$T(b, z, t) = f(z, t) \quad (2.6)$$

$$\left[\frac{\partial T(r,z,t)}{\partial z}\right]_{z=0} = 0 \quad (2.7)$$

$$\left[\frac{\partial T(r,z,t)}{\partial z}\right]_{z=\infty} = 0 \quad (2.8)$$

where k is the thermal diffusivity of the material of the cylinder.

The radial and axial displacement U and W satisfying the uncoupled thermoelastic equations as [2] are

$$\nabla^2 U - \frac{U}{r^2} + (1-2\nu)^{-1}\frac{\partial e}{\partial r} = 2\frac{(1+\nu)}{(1-2\nu)}a_t\frac{\partial T}{\partial r} \quad (2.9)$$

$$\nabla^2 W + (1+2\nu)^{-1}\frac{\partial e}{\partial z} = 2\frac{(1+\nu)}{(1-2\nu)}\frac{\partial T}{\partial z} \quad (2.10)$$

where $e = \frac{\partial U}{\partial r} + \frac{U}{r} + \frac{\partial W}{\partial z}$ is the volume dilation and

$$U = \frac{\partial \phi}{\partial r} \quad (2.11)$$

$$W = \frac{\partial \phi}{\partial z} \quad (2.12)$$

The stress functions are given by

$$\tau_{rz}(a, z, t) = 0, \quad \tau_{rz}(b, z, t) = 0, \quad \tau_{rz}(r, 0, t) = 0 \quad (2.13)$$

$$\sigma_r(a, z, t) = p_1, \ \sigma_r(b, z, t) = -p_0, \ \sigma_z(r, h, t) = 0 \quad (2.14)$$

where p_1 and p_0 are the surface pressures assumed to be uniform over the boundaries of the cylinder. The stress functions are expressed in terms of the displacement components by the following relations:

$$\sigma_r = (\lambda + 2G)\frac{\partial U}{\partial r} + \lambda\left[\frac{U}{r} + \frac{\partial W}{\partial z}\right] \quad (2.15)$$

$$\sigma_z = (\lambda + 2G)\frac{\partial W}{\partial z} + \lambda\left[\frac{\partial U}{\partial r} + \frac{U}{r}\right] \quad (2.16)$$

$$\sigma_\theta = (\lambda + 2G)\frac{U}{r} + \lambda\left[\frac{\partial U}{\partial r} + \frac{\partial W}{\partial z}\right] \quad (2.17)$$

$$\tau_{rz} = G\left[\frac{\partial W}{\partial r} + \frac{\partial U}{\partial z}\right] \quad (2.18)$$

where $\lambda = \left(\frac{2G\nu}{1-2\nu}\right)$ is the lame's constant, G is the shear modulus and U and W are the displacement components. Equations (2.1) to (2.18) constitute the mathematical formulation of the problem under consideration.

3. Solution of the Problem

Applying Fourier cosine transform to the equations (2.3) to (2.6) and using the conditions (2.7), (2.8) one obtains

$$\frac{d^2\overline{T}}{dr^2}+\frac{1}{r}\frac{d\overline{T}}{dr}-p^2\overline{T}=\tfrac{1}{k}\frac{d\overline{T}}{dt} \tag{3.1}$$

$$\overline{T}(r,n,0)=0 \tag{3.2}$$

$$\overline{T}(a,n,t)=0 \tag{3.3}$$

$$\overline{T}(b,n,t)=\overline{f}(n,t) \tag{3.4}$$

where $\overline{T}$ denotes Fourier cosine transform of T , n is Fourier cosine transform parameter and

$p^2=n^2\pi^2$

Applying Laplace transform to the equations (3.1), (3.3), (3.4) and using the condition (3.2) one obtains

$$\frac{d^2\overline{T}^*}{dr^2}+\frac{1}{r}\frac{d\overline{T}^*}{dr}-q^2\overline{T}^*=0 \tag{3.5}$$

where $q^2=p^2+\frac{s}{k}$ (3.6)

$$\overline{T}^*(a,n,s)=0 \tag{3.7}$$

$$\overline{T}^*(b,n,s)=\overline{f}^*(n,s) \tag{3.8}$$

where $\overline{T}^*$ denotes Laplace transform of $\overline{T}$ and s is Laplace transform parameter.
Equation (3.5) is a Bessel's equation whose gives

$$\overline{T}^*(r,n,s)=\mathrm{A}\,\mathrm{I}_0(\mathrm{qr})+\mathrm{B}\,\mathrm{K}_0(\mathrm{qr}) \tag{3.9}$$

where A, B are constants and I_0, K_0 are modified Bessel's functions of first and second kind of order zero respectively.
Using (3.7) and (3.8) in (3.9) one obtains

$$AI_0(qa)+BK_0(qa)=0 \tag{3.10}$$

$$AI_0(qb)+BK_0(qb)=\overline{f}^*(n,s) \tag{3.11}$$

Solving (3.10) and (3.11) one obtains

$$A=\frac{\overline{f}^*(n,s)K_0(qa)}{I_0(qb)K_0(qa)-K_0(qb)I_0(qa)},\quad B=-\frac{\overline{f}^*(n,s)I_0(qa)}{I_0(qb)K_0(qa)-K_0(qb)I_0(qa)}$$

Substituting the values of A and B in (3.9) we get

$$\overline{T}^*(r,n,s)=\overline{f}^*(n,s)\left[\frac{I_0(qr)K_0(qa)-K_0(qr)I_0(qa)}{I_0(qb)K_0(qa)-K_0(qb)I_0(qa)}\right] \tag{3.12}$$

Applying Inverse Laplace transform to the equation (3.12) one obtains

$$\overline{T}(r,n,t)=L^{-1}\left[\overline{f}^*(n,s).\overline{g}^*(s)\right] \tag{3.13}$$

Where $\overline{g}^*(s)=\left[\frac{I_0(qr)K_0(qa)-K_0(qr)I_0(qa)}{I_0(qb)K_0(qa)-K_0(qb)I_0(qa)}\right]$ (3.14)

To Calculate the Inverse Laplace Transform of (3.14):

Applying inverse Laplace transform to the equation (3.14) we get

$$\bar{g}(t) = \frac{1}{2\pi i}\int_{c-i\infty}^{c+i\infty} e^{st}\left[\frac{I_0(qr)K_0(qa) - K_0(qr)I_0(qa)}{I_0(qb)K_0(qa) - K_0(qb)I_0(qa)}\right]ds \qquad (3.14)$$

where c is greater than the real part of the singularities of the integrand.
The integrand is a single valued function of s. The poles of the integrand are at the points

$$s = s_n = -k\left[p^2 + \lambda_m{}^2\right]$$

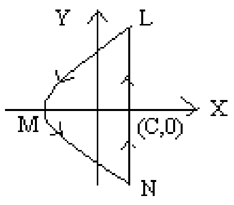
where λ_m are the positive roots of the transcendental equation

$$J_0(\lambda_m r)Y_0(\lambda_m a) - Y_0(\lambda_m r)J_0(\lambda_m a) = 0 \qquad (3.15)$$

The zeros of $I_0(qb)K_0(qa) - K_0(qb)I_0(qa)$ are all real and simple.
The poles of the integrand (3.14) are at

$$s = -k\left[p^2 + \lambda_m{}^2\right] \qquad n = 1,2,3,\ldots$$

Using the contour of figure given below,



The integral (3.14) is equal to 2πi times the sum of the residues at the poles of the integrand.
To find the residue at the point, one requires the result:

$$\left\{s\frac{d}{ds}(I_0(qa)K_0(qb) - K_0(qa)I_0(qb))\right\} = \frac{a}{2kq}\left[(I_0{}'(qa)K_0(qb) - K_0{}'(qa)I_0(qb))\right]$$
$$+\frac{b}{2kq}\left[(I_0(qa)K_0{}'(qb) - K_0(qa)I_0{}'(qb))\right] \qquad (3.16)$$

where $s = -k\left[p^2 + \lambda_m{}^2\right]$, $q^2 = a_n{}^2 + \frac{s}{k}$ and $\frac{I_0(qa)}{I_0(qb)} = \frac{K_0(qa)}{K_0(qb)} = \frac{J_0(\lambda_m a)}{J_0(\lambda_m b)}$

Equation (3.20) reduces to the form

$$(p^2 + \lambda_m{}^2)\,\frac{J_0{}^2(\lambda_m b) - J_0{}^2(\lambda_m a)}{2\lambda_m{}^2 J_0(\lambda_m b)J_0(\lambda_m a)} \qquad (3.17)$$

Also

$$\left(I_0(qa)K_0(qr) - K_0(qa)I_0(qr)\right)\Big|_{s=-k(\lambda_m{}^2+p^2)} = -\frac{\pi}{2}\times\left(J_0(\lambda_m a)Y_0(\lambda_m r) - Y_0(\lambda_m a)J_0(\lambda_m r)\right) \qquad (3.18)$$

Using (3.16) and (3.18) in (3.14) one obtains

$$\bar{g}(t) = -\pi\lambda_m{}^2\,\frac{\left(J_0(\lambda_m a)Y_0(\lambda_m r) - Y_0(\lambda_m a)J_0(\lambda_m r)\right)}{\left(\lambda_m{}^2 + p^2\right)\left(J_0{}^2(\lambda_m b) - J_0{}^2(\lambda_m a)\right)}\times J_0(\lambda_m a)J_0(\lambda_m b)e^{-k(\lambda_m{}^2+p^2)t}$$

Applying convolution theorem to the equation (3.12) we get

$$\bar{T}(r,n,t) = \pi\sum_{m=1}^{\infty}\frac{\lambda_m{}^2 J_0(\lambda_m a)J_0(\lambda_m b)}{\left(\lambda_m{}^2 + p^2\right)\left(J_0{}^2(\lambda_m a) - J_0{}^2(\lambda_m b)\right)}\times\left[J_0(\lambda_m a)Y_0(\lambda_m r) - Y_0(\lambda_m a)J_0(\lambda_m r)\right]$$
$$\times\int_0^t \bar{f}(n,t')e^{-k(\lambda_m{}^2+p^2)(t-t')}dt' \qquad (3.19)$$

Applying inverse Fourier cosine transform to the equation (3.19) we get

$$T(r,z,t)=\sum_{n=1}^{\infty}\cos(pz)\sum_{m=1}^{\infty}\frac{\lambda_m{}^2 J_0(\lambda_m a)J_0(\lambda_m b)}{(\lambda_m{}^2+p^2)(J_0{}^2(\lambda_m a)-J_0{}^2(\lambda_m b))}\times[J_0(\lambda_m a)Y_0(\lambda_m r)-J_0(\lambda_m r)Y_0(\lambda_m a)]$$
$$\times\int_0^t \bar{f}(n,t')e^{-k(\lambda_m{}^2+p^2)(t-t')}\,dt' \qquad (3.20)$$

where m, n are positive integers, λ_m are the positive roots of the transcendental equation $J_0(\lambda_m a)Y_0(\lambda_m b)-J_0(\lambda_m b)Y_0(\lambda_m a)=0$,

$$\bar{f}(n)=\int_0^{\infty} f(z)\cos(pz)\,dz$$

4. Thermoelastic Displacement Functions

Substituting the value of T(r,z,t) from (3.20) in (2.1), one obtains the thermoelastic displacement function ϕ(r, z, t) as

$$\phi(r,z,t)=\left(\frac{1+\nu}{1-\nu}\right)\frac{a_t}{4}\sum_{n=1}^{\infty}r^2\cos(pz)\ \times\sum_{m=1}^{\infty}\frac{\lambda_m{}^2 J_0(\lambda_m a)J_0(\lambda_m b)}{(\lambda_m{}^2+p^2)(J_0{}^2(\lambda_m a)-J_0{}^2(\lambda_m b))}$$
$$\times[J_0(\lambda_m a)Y_0(\lambda_m r)-J_0(\lambda_m r)Y_0(\lambda_m a)]\times\int_0^t \bar{f}(n,t')e^{-k(\lambda_m{}^2+p^2)(t-t')}\,dt' \qquad (4.1)$$

Using (4.1) in (2.11) and (2.12) one obtains the radial and axial displacement U and W as

$$U=\left(\frac{1+\nu}{1-\nu}\right)\frac{a_t}{4}\sum_{n=1}^{\infty}\cos(pz)\times\sum_{m=1}^{\infty}\frac{\lambda_m{}^2 J_0(\lambda_m a)J_0(\lambda_m b)}{(\lambda_m{}^2+p^2)(J_0{}^2(\lambda_m a)-J_0{}^2(\lambda_m b))}$$
$$\times\left[2r(J_0(\lambda_m a)Y_0(\lambda_m r)-J_0(\lambda_m r)Y_0(\lambda_m a))+\lambda_m r^2\left(J_0(\lambda_m a)Y_0{}'(\lambda_m r)-J_0{}'(\lambda_m r)Y_0(\lambda_m a)\right)\right]$$
$$\times\int_0^t \bar{f}(n,t')e^{-k(\lambda_m{}^2+p^2)(t-t')}\,dt' \qquad (4.2)$$

$$W=-\left(\frac{1+\nu}{1-\nu}\right)\frac{a_t}{4}\sum_{n=1}^{\infty}r^2 p\sin(pz)\times\sum_{m=1}^{\infty}\frac{\lambda_m{}^2 J_0(\lambda_m a)J_0(\lambda_m b)}{(\lambda_m{}^2+p^2)(J_0{}^2(\lambda_m a)-J_0{}^2(\lambda_m b))}$$
$$\times[J_0(\lambda_m a)Y_0(\lambda_m r)-J_0(\lambda_m r)Y_0(\lambda_m a)]\times\int_0^t \bar{f}(n,t')e^{-k(\lambda_m{}^2+p^2)(t-t')}\,dt' \qquad (4.3)$$

5. Determination of Stress Functions

Using (4.2) and (4.3) in (2.15) to (2.18), the stress functions are obtained as

$$\sigma_r=(\lambda+2G)\left(\frac{1+\nu}{1-\nu}\right)\frac{a_t}{4}\sum_{n=1}^{\infty}\cos(pz)\ \times\sum_{m=1}^{\infty}\frac{\lambda_m{}^2 J_0(\lambda_m a)J_0(\lambda_m b)}{(\lambda_m{}^2+p^2)(J_0{}^2(\lambda_m a)-J_0{}^2(\lambda_m b))}$$
$$\times\left[2(J_0(\lambda_m a)Y_0(\lambda_m r)-J_0(\lambda_m r)Y_0(\lambda_m a))+4\lambda_m r\left(J_0(\lambda_m a)Y_0{}'(\lambda_m r)-J_0{}'(\lambda_m r)Y_0(\lambda_m a)\right)\right.$$

$$+\lambda_m{}^2 r^2 (J_0(\lambda_m a)Y_0''(\lambda_m r) - J_0''(\lambda_m r)Y_0(\lambda_m a))\Big] \times \int_0^t \overline{f}(n,t')e^{-k(\lambda_m{}^2+p^2)(t-t')}\,dt'$$

$$+\lambda\left(\frac{1+\nu}{1-\nu}\right)\frac{a_t}{4}\sum_{n=1}^{\infty}\cos(pz)\times\sum_{m=1}^{\infty}\frac{\lambda_m{}^2 J_0(\lambda_m a)J_0(\lambda_m b)}{(\lambda_m{}^2+p^2)(J_0{}^2(\lambda_m a)-J_0{}^2(\lambda_m b))}$$

$$\times\Big[2(J_0(\lambda_m a)Y_0(\lambda_m r) - J_0(\lambda_m r)Y_0(\lambda_m a) + \lambda_m r(J_0(\lambda_m a)Y_0'(\lambda_m r) - J_0'(\lambda_m r)Y_0(\lambda_m a))\Big]$$

$$\times\int_0^t \overline{f}(n,t')e^{-k(\lambda_m{}^2+p^2)(t-t')}\,dt'$$

$$-\lambda\left(\frac{1+\nu}{1-\nu}\right)\frac{a_t}{4}\sum_{n=1}^{\infty}r^2p^2\cos(pz)\;\times\sum_{m=1}^{\infty}\frac{\lambda_m{}^2 J_0(\lambda_m a)J_0(\lambda_m b)}{(\lambda_m{}^2+p^2)(J_0{}^2(\lambda_m a)-J_0{}^2(\lambda_m b))}$$

$$\times[J_0(\lambda_m a)Y_0(\lambda_m r) - J_0(\lambda_m r)Y_0(\lambda_m a)]\times\int_0^t \overline{f}(n,t')e^{-k(\lambda_m{}^2+p^2)(t-t')}\,dt' \qquad (5.1)$$

$$\sigma_z = -(\lambda+2G)\left(\frac{1+\nu}{1-\nu}\right)\frac{a_t}{4}\sum_{n=1}^{\infty}r^2p^2\cos(pz)\;\times\sum_{m=1}^{\infty}\frac{\lambda_m{}^2 J_0(\lambda_m a)J_0(\lambda_m b)}{(\lambda_m{}^2+p^2)(J_0{}^2(\lambda_m a)-J_0{}^2(\lambda_m b))}$$

$$\times[J_0(\lambda_m a)Y_0(\lambda_m r) - J_0(\lambda_m r)Y_0(\lambda_m a)]\times\int_0^t \overline{f}(n,t')e^{-k(\lambda_m{}^2+p^2)(t-t')}\,dt'$$

$$+\lambda\left(\frac{1+\nu}{1-\nu}\right)\frac{a_t}{4}\sum_{n=1}^{\infty}\cos(pz)\times\sum_{m=1}^{\infty}\frac{\lambda_m{}^2 J_0(\lambda_m a)J_0(\lambda_m b)}{(\lambda_m{}^2+p^2)(J_0{}^2(\lambda_m a)-J_0{}^2(\lambda_m b))}$$

$$\times\Big[2(J_0(\lambda_m a)Y_0(\lambda_m r) - J_0(\lambda_m r)Y_0(\lambda_m a)) + 4\lambda_m r\left(J_0(\lambda_m a)Y_0'(\lambda_m r) - J_0'(\lambda_m r)Y_0(\lambda_m a)\right)$$

$$+\lambda_m{}^2 r^2 (J_0(\lambda_m a)Y_0''(\lambda_m r) - J_0''(\lambda_m r)Y_0(\lambda_m a))\Big]\;\times\int_0^t \overline{f}(n,t')e^{-k(\lambda_m{}^2+p^2)(t-t')}\,dt'$$

$$+\lambda\left(\frac{1+\nu}{1-\nu}\right)\frac{a_t}{4}\sum_{n=1}^{\infty}\cos(pz)\;\times\sum_{m=1}^{\infty}\frac{\lambda_m{}^2 J_0(\lambda_m a)J_0(\lambda_m b)}{(\lambda_m{}^2+p^2)(J_0{}^2(\lambda_m a)-J_0{}^2(\lambda_m b))}$$

$$\times\Big[2(J_0(\lambda_m a)Y_0(\lambda_m r) - J_0(\lambda_m r)Y_0(\lambda_m a)) + \lambda_m r(J_0(\lambda_m a)Y_0'(\lambda_m r) - J_0'(\lambda_m r)Y_0(\lambda_m a))\Big]$$

$$\times\int_0^t \overline{f}(n,t')e^{-k(\lambda_m{}^2+p^2)(t-t')}\,dt' \qquad (5.2)$$

$$\sigma_\theta = (\lambda+2G)\left(\frac{1+\nu}{1-\nu}\right)\frac{a_t}{4}\sum_{n=1}^{\infty}\cos(pz)\;\times\sum_{m=1}^{\infty}\frac{\lambda_m{}^2 J_0(\lambda_m a)J_0(\lambda_m b)}{(\lambda_m{}^2+p^2)(J_0{}^2(\lambda_m a)-J_0{}^2(\lambda_m b))}$$

$$\times\left[2(J_0(\lambda_m a)Y_0(\lambda_m r)-J_0(\lambda_m r)Y_0(\lambda_m a))+\lambda_m r(J_0(\lambda_m a)Y_0'(\lambda_m r)-J_0'(\lambda_m r)Y_0(\lambda_m a))\right]$$

$$\times\int_0^t \overline{f}(n,t')e^{-k(\lambda_m{}^2+p^2)(t-t')}\,dt'$$

$$+\lambda\left(\frac{1+\nu}{1-\nu}\right)\frac{a_t}{4}\sum_{n=1}^{\infty}\cos(pz)\times\sum_{m=1}^{\infty}\frac{\lambda_m{}^2 J_0(\lambda_m a)J_0(\lambda_m b)}{(\lambda_m{}^2+p^2)(J_0{}^2(\lambda_m a)-J_0{}^2(\lambda_m b))}$$

$$\times\left[2(J_0(\lambda_m a)Y_0(\lambda_m r)-J_0(\lambda_m r)Y_0(\lambda_m a))+4\lambda_m r\left(J_0(\lambda_m a)Y_0'(\lambda_m r)-J_0'(\lambda_m r)Y_0(\lambda_m a)\right)\right.$$

$$\left.+\lambda_m{}^2 r^2(J_0(\lambda_m a)Y_0''(\lambda_m r)-J_0''(\lambda_m r)Y_0(\lambda_m a))\right]\times\int_0^t \overline{f}(n,t')e^{-k(\lambda_m{}^2+p^2)(t-t')}\,dt'$$

$$-\lambda\left(\frac{1+\nu}{1-\nu}\right)\frac{a_t}{4}\sum_{n=1}^{\infty}r^2p^2\cos(pz)\times\sum_{m=1}^{\infty}\frac{\lambda_m{}^2 J_0(\lambda_m a)J_0(\lambda_m b)}{(\lambda_m{}^2+p^2)(J_0{}^2(\lambda_m a)-J_0{}^2(\lambda_m b))}$$

$$\times[J_0(\lambda_m a)Y_0(\lambda_m r)-J_0(\lambda_m r)Y_0(\lambda_m a)]\times\int_0^t \overline{f}(n,t')e^{-k(\lambda_m{}^2+p^2)(t-t')}\,dt' \qquad (5.3)$$

$$\tau_{rz}=-G\left(\frac{1+\nu}{1-\nu}\right)\frac{a_t}{2}\sum_{n=1}^{\infty}p\sin(pz)\times\sum_{m=1}^{\infty}\frac{\lambda_m{}^2 J_0(\lambda_m a)J_0(\lambda_m b)}{(\lambda_m{}^2+p^2)(J_0{}^2(\lambda_m a)-J_0{}^2(\lambda_m b))}$$

$$\times\left[2r(J_0(\lambda_m a)Y_0(\lambda_m r)-J_0(\lambda_m r)Y_0(\lambda_m a))+\lambda_m r^2\left(J_0(\lambda_m a)Y_0'(\lambda_m r)-J_0'(\lambda_m r)Y_0(\lambda_m a)\right)\right.$$

$$\times\int_0^t \overline{f}(n,t')e^{-k(\lambda_m{}^2+p^2)(t-t')}\,dt' \qquad (5.4)$$

6. Special Case

Set $f(z,t)=(1-e^{-t})e^{-z^2}(b-a)$ (6.1)

Applying Fourier cosine transform to the equation (6.1) one obtains

$$\overline{f}(n,t)=\int_0^{\infty}(1-e^{-t})e^{-z^2}(b-a)\cos(pz)dz \quad =\frac{(1-e^{-t})(b-a)\sqrt{\pi}}{2}\left[e^{-p^{2/4}}\right] \qquad (6.2)$$

Substituting the value of $\overline{f}(n,t)$ from (6.2) in the equations (3.20) one obtains

$$T(r,z,t)=\frac{(b-a)\sqrt{\pi}}{2}\sum_{n=1}^{\infty}\left[e^{-p^2/4}\right]\cos(pz)\times\sum_{m=1}^{\infty}\frac{\lambda_m{}^2 J_0(\lambda_m a)J_0(\lambda_m b)}{(\lambda_m{}^2+p^2)(J_0{}^2(\lambda_m a)-J_0{}^2(\lambda_m b))}$$

$$\times[J_0(\lambda_m a)Y_0(\lambda_m r)-J_0(\lambda_m r)Y_0(\lambda_m a)]\times\int_0^t(1-e^{-t'})e^{-k(\lambda_m{}^2+p^2)(t-t')}\,dt' \qquad (6.3)$$

7. Numerical Results

Set $\alpha = \frac{(b-a)\sqrt{\pi}}{2}$, a = 1 m, b= 2 m, t =1 sec, k = 0.86 and λ_m are the roots of the transcendental equation $J_0(\lambda_m a)Y_0(\lambda_m b) - J_0(\lambda_m b)Y_0(\lambda_m a) = 0$
as [16] in equation (6.3) one obtains

$$\frac{T(r,z,t)}{\alpha} = \sum_{n=1}^{\infty}\left(e^{-p^2/4}\right)\cos(pz) \times \left\{\sum_{m=1}^{\infty} \frac{\lambda_m{}^2 J_0(\lambda_m)J_0(2\lambda_m)}{(\lambda_m{}^2+p^2)(J_0{}^2(\lambda_m)-J_0{}^2(2\lambda_m))}\right.$$

$$\left.\times[J_0(\lambda_m)Y_0(\lambda_m r) - J_0(\lambda_m r)Y_0(\lambda_m)]\right\} \times \int_0^1 (1-e^{-t'})e^{-0.86(\lambda_m{}^2+p^2)(1-t')}\,dt' \qquad (7.1)$$

8. Statement of the Problem-II

Consider semi-infinite hollow cylinder occupying the space D : $a \le r \le b$, $0 \le z \le \infty$. The thermoelastic displacement function as [1] is governed by the Poisson's equation

$$\nabla^2\phi = \left(\frac{1+\nu}{1-\nu}\right)a_t T \qquad (8.1)$$

with $\phi = 0$ at $r = a$ and $r = b$ (8.2)

where $\nabla^2 = \frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r} + \frac{\partial^2}{\partial z^2}$

ν and a_t are the Poisson's ratio and the linear coefficient of thermal expansion of the material of the cylinder and T is the temperature of the cylinder satisfying the differential equation

$$\frac{\partial^2 T}{\partial r^2} + \frac{1}{r}\frac{\partial T}{\partial r} + \frac{\partial^2 T}{\partial z^2} = \frac{1}{k}\frac{\partial T}{\partial t} \qquad (8.3)$$

subject to the initial conditions

$$T(r,z,0) = 0 \qquad (8.4)$$

The boundary conditions are

$$T(a,z,t) = u(z,t) \qquad (8.5)$$

$$T(b,z,t) = f(z,t) \qquad (8.6)$$

$$\left[\frac{\partial T(r,z,t)}{\partial z}\right]_{z=0} = 0 \qquad (8.7)$$

$$\left[\frac{\partial T(r,z,t)}{\partial z}\right]_{z=\infty} = 0 \qquad (8.8)$$

where k is the thermal diffusivity of the material of the cylinder.
The radial and axial displacement U and W satisfying the uncoupled thermoelastic equations as [2] are

$$\nabla^2 U - \frac{U}{r^2} + (1-2\nu)^{-1}\frac{\partial e}{\partial r} = 2\frac{(1+\nu)}{(1-2\nu)}a_t\frac{\partial T}{\partial r} \qquad (8.9)$$

$$\nabla^2 W + (1+2\nu)^{-1}\frac{\partial e}{\partial z} = 2\frac{(1+\nu)}{(1-2\nu)}\frac{\partial T}{\partial z} \qquad (8.10)$$

where $e = \frac{\partial U}{\partial r} + \frac{U}{r} + \frac{\partial W}{\partial z}$ is the volume dilation and

$$U = \frac{\partial \phi}{\partial r} \quad (8.11)$$

$$W = \frac{\partial \phi}{\partial z} \quad (8.12)$$

The stress functions are given by

$$\tau_{rz}(a,z,t) = 0, \quad \tau_{rz}(b,z,t) = 0, \quad \tau_{rz}(r,0,t) = 0 \quad (8.13)$$

$$\sigma_r(a,z,t) = p_1, \ \sigma_r(b,z,t) = -p_0, \ \sigma_z(r,h,t) = 0 \quad (8.14)$$

where p_1 and p_0 are the surface pressures assumed to be uniform over the boundaries of the cylinder. The stress functions are expressed in terms of the displacement components by the following relations:

$$\sigma_r = (\lambda + 2G)\frac{\partial U}{\partial r} + \lambda\left[\frac{U}{r} + \frac{\partial W}{\partial z}\right] \quad (8.15)$$

$$\sigma_z = (\lambda + 2G)\frac{\partial W}{\partial z} + \lambda\left[\frac{\partial U}{\partial r} + \frac{U}{r}\right] \quad (8.16)$$

$$\sigma_\theta = (\lambda + 2G)\frac{U}{r} + \lambda\left[\frac{\partial U}{\partial r} + \frac{\partial W}{\partial z}\right] \quad (8.17)$$

$$\tau_{rz} = G\left[\frac{\partial W}{\partial r} + \frac{\partial U}{\partial z}\right] \quad (8.18)$$

where $\lambda = \left(\frac{2G\nu}{1-2\nu}\right)$ is Lame's constant, G is the shear modulus and U and W are the displacement components. The equations (8.1) to (8.18) constitute the mathematical formulation of the problem under consideration.

9. Solution of the Problem

Applying Fourier cosine transform to the equations (8.3), (8.4), (8.5), (8.6) and using (8.7), (8.8) one obtains

$$\frac{d^2\overline{T}}{dr^2} + \frac{1}{r}\frac{d\overline{T}}{dr} - p^2\overline{T} = \frac{1}{k}\frac{d\overline{T}}{dt} \quad (9.1)$$

$$\overline{T}(r,n,0) = 0 \quad (9.2)$$

$$\overline{T}(a,n,t) = \overline{u}(n,t) \quad (9.3)$$

$$\overline{T}(b,n,t) = \overline{f}(n,t) \quad (9.4)$$

where $\overline{T}$ denotes Fourier cosine transform of T and n is Fourier cosine transform parameter.
Applying Laplace transform to the equations (9.1), (9.3), (9.4) and using (9.2) one obtains

$$\frac{d^2\overline{T}^*}{dr^2} + \frac{1}{r}\frac{d\overline{T}^*}{dr} - q^2\overline{T}^* = 0 \quad (9.5)$$

where $q^2 = p^2 + \frac{s}{k}$ (9.6)

$$\overline{T}^*(a,n,s) = \overline{u}^*(n,s) \quad (9.7)$$

$$\overline{T}^*(b,n,s) = \overline{f}^*(n,s) \quad (9.8)$$

where $\overline{T}^*$ denotes the Laplace transform of $\overline{T}$ and s is a Laplace transform parameter.
The equation (9.5) is a Bessel's equation whose solution gives

$$\overline{T}^*(r,n,s) = A\, I_0(qr) + B\, K_0(qr) \quad (9.9)$$

where A, B are constants and I_0, K_0 are modified Bessel's functions of first and second kind of order zero respectively.
Using (9.7) and (9.8) in (9.9) we get

$$AI_0(qa) + BK_0(qa) = \bar{u}^*(n,s) \tag{9.10}$$

$$AI_0(qb) + BK_0(qb) = \bar{f}^*(n,s) \tag{9.11}$$

Solving (9.10) and (9.11) one obtains

$$A = \frac{\bar{f}^*(n)K_0(qa)}{I_0(qb)K_0(qa) - K_0(qb)I_0(qa)} - \frac{\bar{u}^*(n)K_0(qb)}{I_0(qb)K_0(qa) - K_0(qb)I_0(qa)}$$

$$B = -\frac{\bar{f}^*(n,s)I_0(qa)}{I_0(qb)K_0(qa) - K_0(qb)I_0(qa)} + \frac{\bar{u}^*(n,s)I_0(qb)}{I_0(qb)K_0(qa) - K_0(qb)I_0(qa)}$$

Substituting the values of A and B in (9.9) one obtains

$$\bar{T}^*(r,n,s) = \bar{f}^*(n,s)\left[\frac{I_0(qr)K_0(qa) - K_0(qr)I_0(qa)}{I_0(qb)K_0(qa) - K_0(qb)I_0(qa)}\right] - \bar{u}^*(n,s)\left[\frac{I_0(qr)K_0(qb) - K_0(qr)I_0(qb)}{I_0(qb)K_0(qa) - K_0(qb)I_0(qa)}\right] \tag{9.12}$$

Applying Inverse Laplace transform and inverse Fourier cosine transform to (.9.12) one obtains the expressions for the temperature distribution T(r,z,t) as

$$T(r,z,t) = \sum_{n=1}^{\infty}\cos(pz) \times \sum_{m=1}^{\infty}\frac{\lambda_m{}^2 J_0(\lambda_m a)J_0(\lambda_m b)}{(\lambda_m{}^2 + p^2)(J_0{}^2(\lambda_m a) - J_0{}^2(\lambda_m b))}$$

$$\times[J_0(\lambda_m a)Y_0(\lambda_m r) - J_0(\lambda_m r)Y_0(\lambda_m a)] \times \int_0^t \bar{f}(n,t')e^{-k(\lambda_m{}^2 + p^2)(t-t')}\,dt'$$

$$-\sum_{n=1}^{\infty}\cos(pz\sum_{m=1}^{\infty}\frac{\lambda_m{}^2 J_0{}^2(\lambda_m b)}{(\lambda_m{}^2 + p^2)[J_0{}^2(\lambda_m a) - J_0{}^2(\lambda_m b)]}$$

$$\times[J_0(\lambda_m b)Y_0(\lambda_m r) - J_0(\lambda_m r)Y_0(\lambda_m b)] \times \int_0^t \bar{u}(n,t')e^{-k(\lambda_m{}^2 + p^2)(t-t')}\,dt' \tag{9.13}$$

where m, n are positive integers, λ_m are the positive roots of the transcendental equation

$$J_0(\lambda_m a)Y_0(\lambda_m b) - J_0(\lambda_m b)Y_0(\lambda_m a) = 0$$

$$\bar{f}(n) = \int_0^{\infty} f(z)\cos(pz)\,dz\,, \quad \bar{u}(n) = \int_0^{\infty} u(z)\cos(pz)\,dz$$

10. Determination of Thermoelastic Displacement

Substituting the value of T(r,z,t) from (9.13) in (8.1), one obtains the thermoelastic displacement function ϕ(r, z,t) as

$$\phi(r,z,t) = \left(\frac{1+\nu}{1-\nu}\right)\frac{a_t}{4}\sum_{n=1}^{\infty} r^2\cos(pz) \times \sum_{m=1}^{\infty}\frac{\lambda_m{}^2 J_0(\lambda_m a)J_0(\lambda_m b)}{(\lambda_m{}^2 + p^2)(J_0{}^2(\lambda_m a) - J_0{}^2(\lambda_m b))}$$

$$\times[J_0(\lambda_m a)Y_0(\lambda_m r) - J_0(\lambda_m r)Y_0(\lambda_m a)] \times \int_0^t \bar{f}(n,t')e^{-k(\lambda_m{}^2 + p^2)(t-t')}\,dt'$$

$$-\left(\frac{1+\nu}{1-\nu}\right)\frac{a_t}{4}\sum_{n=1}^{\infty} r^2\cos(pz)\times\sum_{m=1}^{\infty}\frac{\lambda_m{}^2 J_0{}^2(\lambda_m b)}{(\lambda_m{}^2+p^2)[J_0{}^2(\lambda_m a)-J_0{}^2(\lambda_m b)]}$$

$$\times[J_0(\lambda_m b)Y_0(\lambda_m r)-J_0(\lambda_m r)Y_0(\lambda_m b)]\times\int_0^t \bar{u}(n,t')e^{-k(\lambda_m{}^2+p^2)(t-t')}\,dt' \tag{10.1}$$

Using (10.1) in (8.11) and (8.12) one obtains the radial and axial displacement U and as

$$U=\left(\frac{1+\nu}{1-\nu}\right)\frac{a_t}{4}\sum_{n=1}^{\infty}\cos(pz)\times\sum_{m=1}^{\infty}\frac{\lambda_m{}^2 J_0(\lambda_m a)J_0(\lambda_m b)}{(\lambda_m{}^2+p^2)(J_0{}^2(\lambda_m a)-J_0{}^2(\lambda_m b))}$$

$$\times\left[2r(J_0(\lambda_m a)Y_0(\lambda_m r)-J_0(\lambda_m r)Y_0(\lambda_m a))+\lambda_m r^2\left(J_0(\lambda_m a)Y_0{}'(\lambda_m r)-J_0{}'(\lambda_m r)Y_0(\lambda_m a)\right)\right.$$

$$\times\int_0^t \bar{f}(n,t')e^{-k(\lambda_m{}^2+p^2)(t-t')}\,dt'$$

$$-\left(\frac{1+\nu}{1-\nu}\right)\frac{a_t}{4}\sum_{n=1}^{\infty}\cos(pz)\times\sum_{m=1}^{\infty}\frac{\lambda_m{}^2 J_0{}^2(\lambda_m b)}{(\lambda_m{}^2+p^2)(J_0{}^2(\lambda_m a)-J_0{}^2(\lambda_m b))}$$

$$\times\left[2r(J_0(\lambda_m b)Y_0(\lambda_m r)-J_0(\lambda_m r)Y_0(\lambda_m b)+\lambda_m r^2\left(J_0(\lambda_m b)Y_0{}'(\lambda_m r)-J_0{}'(\lambda_m r)Y_0(\lambda_m b)\right)\right.$$

$$\times\int_0^t \bar{u}(n,t')e^{-k(\lambda_m{}^2+p_n{}^2)(t-t')}\,dt' \tag{10.2}$$

$$W=-\left(\frac{1+\nu}{1-\nu}\right)\frac{a_t}{4}\sum_{n=1}^{\infty} r^2 p\sin(pz)\times\sum_{m=1}^{\infty}\frac{\lambda_m{}^2 J_0(\lambda_m a)J_0(\lambda_m b)}{(\lambda_m{}^2+p^2)(J_0{}^2(\lambda_m a)-J_0{}^2(\lambda_m b))}$$

$$\times[J_0(\lambda_m a)Y_0(\lambda_m r)-J_0(\lambda_m r)Y_0(\lambda_m a)]\times\int_0^t \bar{f}(n,t')e^{-k(\lambda_m{}^2+p^2)(t-t')}\,dt'$$

$$+\left(\frac{1+\nu}{1-\nu}\right)\frac{a_t}{4}\sum_{n=1}^{\infty} r^2 p\sin(pz)\times\sum_{m=1}^{\infty}\frac{\lambda_m{}^2 J_0{}^2(\lambda_m b)}{(\lambda_m{}^2+p^2)[J_0{}^2(\lambda_m a)-J_0{}^2(\lambda_m b)]}$$

$$\times[J_0(\lambda_m b)Y_0(\lambda_m r)-J_0(\lambda_m r)Y_0(\lambda_m b)]\times\int_0^t \bar{u}(n,t')e^{-k(\lambda_m{}^2+p^2)(t-t')}\,dt' \tag{10.3}$$

11. Determination of Stress Functions

Using (10.2) and (10.3) in (8.15) to (8.18) the stress functions are obtained as

$$\sigma_r=(\lambda+2G)\left(\frac{1+\nu}{1-\nu}\right)\frac{a_t}{4}\sum_{n=1}^{\infty}\cos(pz)\times\sum_{m=1}^{\infty}\frac{\lambda_m{}^2 J_0(\lambda_m a)J_0(\lambda_m b)}{(\lambda_m{}^2+p^2)(J_0{}^2(\lambda_m a)-J_0{}^2(\lambda_m b))}$$

$$\times\left[2(J_0(\lambda_m a)Y_0(\lambda_m r)-J_0(\lambda_m r)Y_0(\lambda_m a))+4\lambda_m r\left(J_0(\lambda_m a)Y_0{}'(\lambda_m r)-J_0{}'(\lambda_m r)Y_0(\lambda_m a)\right)\right.$$

$$+\lambda_m{}^2 r^2 (J_0(\lambda_m a) Y_0{}''(\lambda_m r) - J_0{}''(\lambda_m r) Y_0(\lambda_m a))\Big] \times \int_0^t \overline{f}(n,t') e^{-k(\lambda_m{}^2+p^2)(t-t')}\, dt'$$

$$+\lambda\left(\tfrac{1+\nu}{1-\nu}\right)\frac{a_t}{4}\sum_{n=1}^{\infty}\cos(pz) \times \sum_{m=1}^{\infty}\frac{\lambda_m{}^2 J_0(\lambda_m a) J_0(\lambda_m b)}{(\lambda_m{}^2+p^2)(J_0{}^2(\lambda_m a) - J_0{}^2(\lambda_m b))}$$

$$\times\Big[2(J_0(\lambda_m a)Y_0(\lambda_m r) - J_0(\lambda_m r)Y_0(\lambda_m a)) + \lambda_m r (J_0(\lambda_m a) Y_0{}'(\lambda_m r) - J_0{}'(\lambda_m r) Y_0(\lambda_m a))\Big]$$

$$\times\int_0^t \overline{f}(n,t') e^{-k(\lambda_m{}^2+p^2)(t-t')}\, dt'$$

$$-\lambda\left(\tfrac{1+\nu}{1-\nu}\right)\frac{a_t}{4}\sum_{n=1} r^2 p^2 \cos(pz) \times \sum_{m=1}^{\infty}\frac{\lambda_m{}^2 J_0(\lambda_m a) J_0(\lambda_m b)}{(\lambda_m{}^2+p^2)(J_0{}^2(\lambda_m a) - J_0{}^2(\lambda_m b))}$$

$$\times[J_0(\lambda_m a)Y_0(\lambda_m r) - J_0(\lambda_m r)Y_0(\lambda_m a)] \times \int_0^t \overline{f}(n,t') e^{-k(\lambda_m{}^2+p^2)(t-t')}\, dt'$$

$$-(\lambda+2G)\left(\tfrac{1+\nu}{1-\nu}\right)\frac{a_t}{4}\sum_{n=1}^{\infty}\cos(pz) \times \sum_{m=1}^{\infty}\frac{\lambda_m{}^2 J_0{}^2(\lambda_m b)}{(\lambda_m{}^2+p^2)(J_0{}^2(\lambda_m a) - J_0{}^2(\lambda_m b))}$$

$$\times\Big[2(J_0(\lambda_m b)Y_0(\lambda_m r) - J_0(\lambda_m r)Y_0(\lambda_m b)) + 4\lambda_m r\left(J_0(\lambda_m b) Y_0{}'(\lambda_m r) - J_0{}'(\lambda_m r) Y_0(\lambda_m b)\right)$$

$$+\lambda_m{}^2 r^2 (J_0(\lambda_m b) Y_0{}'(\lambda_m r) - J_0{}'(\lambda_m r) Y_0(\lambda_m b))\Big] \times \int_0^t \overline{u}(n,t') e^{-k(\lambda_m{}^2+p^2)(t-t')}\, dt'$$

$$-\lambda\left(\tfrac{1+\nu}{1-\nu}\right)\frac{a_t}{4}\sum_{n=1}^{\infty}\cos(pz) \times \sum_{m=1}^{\infty}\frac{\lambda_m{}^2 J_0{}^2(\lambda_m b)}{(\lambda_m{}^2+p^2)(J_0{}^2(\lambda_m a) - J_0{}^2(\lambda_m b))}$$

$$\times\Big[2(J_0(\lambda_m b)Y_0(\lambda_m r) - J_0(\lambda_m r)Y_0(\lambda_m b)) + \lambda_m r (J_0(\lambda_m b) Y_0{}'(\lambda_m r) - J_0{}'(\lambda_m r) Y_0(\lambda_m b))\Big]$$

$$\times\int_0^t \overline{u}(n,t') e^{-k(\lambda_m{}^2+p^2)(t-t')}\, dt'$$

$$+\lambda\left(\tfrac{1+\nu}{1-\nu}\right)\frac{a_t}{4}\sum_{n=1}^{\infty} r^2 p^2 \cos(pz) \times \sum_{m=1}^{\infty}\frac{\lambda_m{}^2 J_0{}^2(\lambda_m b)}{(\lambda_m{}^2+p_n{}^2)[J_0{}^2(\lambda_m a) - J_0{}^2(\lambda_m b)]}$$

$$\times[J_0(\lambda_m b)Y_0(\lambda_m r) - J_0(\lambda_m r)Y_0(\lambda_m b)] \times \int_0^t \overline{u}(n,t') e^{-2k(\lambda_m{}^2+p^2)(t-t')}\, dt' \qquad (11.1)$$

$$\sigma_z = -(\lambda+2G)\left(\tfrac{1+\nu}{1-\nu}\right)\frac{a_t}{4}\sum_{n=1}^{\infty} r^2 p^2 \cos(pz) \times \sum_{m=1}^{\infty}\frac{\lambda_m{}^2 J_0(\lambda_m a) J_0(\lambda_m b)}{(\lambda_m{}^2+p^2)(J_0{}^2(\lambda_m a) - J_0{}^2(\lambda_m b))}$$

$$\times[J_0(\lambda_m a)Y_0(\lambda_m r)-J_0(\lambda_m r)Y_0(\lambda_m a)]\times\int_0^t \overline{f}(n,t')e^{-k(\lambda_m{}^2+p^2)(t-t')}\,dt'$$

$$+\lambda\left(\frac{1+\nu}{1-\nu}\right)\frac{a_t}{4}\sum_{n=1}^{\infty}\cos(pz)\times\sum_{m=1}^{\infty}\frac{\lambda_m{}^2 J_0(\lambda_m a)J_0(\lambda_m b)}{(\lambda_m{}^2+p^2)(J_0{}^2(\lambda_m a)-J_0{}^2(\lambda_m b))}$$

$$\times\Big[2(J_0(\lambda_m a)Y_0(\lambda_m r)-J_0(\lambda_m r)Y_0(\lambda_m a))+4\lambda_m r\Big(J_0(\lambda_m a)Y_0{}'(\lambda_m r)-J_0{}'(\lambda_m r)Y_0(\lambda_m a)\Big)$$

$$+\lambda_m{}^2 r^2(J_0(\lambda_m a)Y_0{}''(\lambda_m r)-J_0{}''(\lambda_m r)Y_0(\lambda_m a))\Big]\times\int_0^t \overline{f}(n,t')e^{-k(\lambda_m{}^2+p^2)(t-t')}\,dt'$$

$$+\lambda\left(\frac{1+\nu}{1-\nu}\right)\frac{a_t}{4}\sum_{n=1}^{\infty}\cos(pz)\times\sum_{m=1}^{\infty}\frac{\lambda_m{}^2 J_0(\lambda_m a)J_0(\lambda_m b)}{(\lambda_m{}^2+p^2)(J_0{}^2(\lambda_m a)-J_0{}^2(\lambda_m b))}$$

$$\times\Big[2(J_0(\lambda_m a)Y_0(\lambda_m r)-J_0(\lambda_m r)Y_0(\lambda_m a))+\lambda_m r(J_0(\lambda_m a)Y_0{}'(\lambda_m r)-J_0{}'(\lambda_m r)Y_0(\lambda_m a))\Big]$$

$$\times\int_0^t \overline{f}(n,t')e^{-k(\lambda_m{}^2+p^2)(t-t')}\,dt'$$

$$+\lambda\left(\frac{1+\nu}{1-\nu}\right)\frac{a_t}{4}\sum_{n=1}^{\infty}r^2p^2\cos(pz)\times\sum_{m=1}^{\infty}\frac{\lambda_m{}^2 J_0{}^2(\lambda_m b)}{(\lambda_m{}^2+p^2)[J_0{}^2(\lambda_m a)-J_0{}^2(\lambda_m b)]}$$

$$\times[J_0(\lambda_m b)Y_0(\lambda_m r)-J_0(\lambda_m r)Y_0(\lambda_m b)]\times\int_0^t \overline{u}(n,t')e^{-2k(\lambda_m{}^2+p^2)(t-t')}\,dt'$$

$$-(\lambda+2G)\left(\frac{1+\nu}{1-\nu}\right)\frac{a_t}{4}\sum_{n=1}^{\infty}\cos(pz)\times\sum_{m=1}^{\infty}\frac{\lambda_m{}^2 J_0{}^2(\lambda_m b)}{(\lambda_m{}^2+p^2)(J_0{}^2(\lambda_m a)-J_0{}^2(\lambda_m b))}$$

$$\times\Big[2(J_0(\lambda_m b)Y_0(\lambda_m r)-J_0(\lambda_m r)Y_0(\lambda_m b))+4\lambda_m r\Big(J_0(\lambda_m b)Y_0{}'(\lambda_m r)-J_0{}'(\lambda_m r)Y_0(\lambda_m b)\Big)$$

$$+\lambda_m{}^2 r^2(J_0(\lambda_m b)Y_0{}'(\lambda_m r)-J_0{}'(\lambda_m r)Y_0(\lambda_m b))\Big]\times\int_0^t \overline{u}(n,t')e^{-k(\lambda_m{}^2+p^2)(t-t')}\,dt'$$

$$-\lambda\left(\frac{1+\nu}{1-\nu}\right)\frac{a_t}{4}\sum_{n=1}^{\infty}\cos(pz)\times\sum_{m=1}^{\infty}\frac{\lambda_m{}^2 J_0{}^2(\lambda_m b)}{(\lambda_m{}^2+p_n{}^2)(J_0{}^2(\lambda_m a)-J_0{}^2(\lambda_m b))}$$

$$\times\Big[2(J_0(\lambda_m b)Y_0(\lambda_m r)-J_0(\lambda_m r)Y_0(\lambda_m b))+\lambda_m r(J_0(\lambda_m b)Y_0{}'(\lambda_m r)-J_0{}'(\lambda_m r)Y_0(\lambda_m b))\Big]$$

$$\times\int_0^t \overline{u}(n,t')e^{-k(\lambda_m{}^2+p^2)(t-t')}\,dt' \qquad (11.2)$$

$$\sigma_\theta=(\lambda+2G)\left(\frac{1+\nu}{1-\nu}\right)\frac{a_t}{4}\sum_{n=1}^{\infty}\cos(pz)\times\sum_{m=1}^{\infty}\frac{\lambda_m{}^2 J_0(\lambda_m a)J_0(\lambda_m b)}{(\lambda_m{}^2+p^2)(J_0{}^2(\lambda_m a)-J_0{}^2(\lambda_m b))}$$

$$\times\left[2(J_0(\lambda_m a)Y_0(\lambda_m r)-J_0(\lambda_m r)Y_0(\lambda_m a))+\lambda_m r(J_0(\lambda_m a)Y_0{}'(\lambda_m r)-J_0{}'(\lambda_m r)Y_0(\lambda_m a))\right]$$

$$\times\int_0^t \overline{f}(n,t')e^{-k(\lambda_m{}^2+p^2)(t-t')}\,dt'$$

$$+\lambda\left(\tfrac{1+\nu}{1-\nu}\right)\frac{a_t}{4}\sum_{n=1}^{\infty}\cos(pz)\times\sum_{m=1}^{\infty}\frac{\lambda_m{}^2J_0(\lambda_m a)J_0(\lambda_m b)}{(\lambda_m{}^2+p^2)(J_0{}^2(\lambda_m a)-J_0{}^2(\lambda_m b))}$$

$$\times\left[2(J_0(\lambda_m a)Y_0(\lambda_m r)-J_0(\lambda_m r)Y_0(\lambda_m a))+4\lambda_m r\left(J_0(\lambda_m a)Y_0{}'(\lambda_m r)-J_0{}'(\lambda_m r)Y_0(\lambda_m a)\right)\right.$$

$$\left.+\lambda_m{}^2r^2(J_0(\lambda_m a)Y_0{}''(\lambda_m r)-J_0{}''(\lambda_m r)Y_0(\lambda_m a))\right]\times\int_0^t \overline{f}(n,t')e^{-k(\lambda_m{}^2+p^2)(t-t')}\,dt'$$

$$-\lambda\left(\tfrac{1+\nu}{1-\nu}\right)\frac{a_t}{4}\sum_{n=1}^{\infty}r^2p^2\cos(pz)\times\sum_{m=1}^{\infty}\frac{\lambda_m{}^2J_0(\lambda_m a)J_0(\lambda_m b)}{(\lambda_m{}^2+p^2)(J_0{}^2(\lambda_m a)-J_0{}^2(\lambda_m b))}$$

$$\times[J_0(\lambda_m a)Y_0(\lambda_m r)-J_0(\lambda_m r)Y_0(\lambda_m a)]\times\int_0^t \overline{f}(n,t')e^{-k(\lambda_m{}^2+p^2)(t-t')}\,dt'$$

$$-(\lambda+2G)\left(\tfrac{1+\nu}{1-\nu}\right)\frac{a_t}{4}\sum_{n=1}^{\infty}\cos(pz)\times\sum_{m=1}^{\infty}\frac{\lambda_m{}^2J_0{}^2(\lambda_m b)}{(\lambda_m{}^2+p^2)(J_0{}^2(\lambda_m a)-J_0{}^2(\lambda_m b))}$$

$$\times\left[2(J_0(\lambda_m b)Y_0(\lambda_m r)-J_0(\lambda_m r)Y_0(\lambda_m b))+\lambda_m r(J_0(\lambda_m b)Y_0{}'(\lambda_m r)-J_0{}'(\lambda_m r)Y_0(\lambda_m b))\right]$$

$$\times\int_0^t \overline{u}(n,t')e^{-k(\lambda_m{}^2+p^2)(t-t')}\,dt'$$

$$-\lambda\left(\tfrac{1+\nu}{1-\nu}\right)\frac{a_t}{4}\sum_{n=1}^{\infty}\cos(pz)\times\sum_{m=1}^{\infty}\frac{\lambda_m{}^2J_0{}^2(\lambda_m b)}{(\lambda_m{}^2+p^2)(J_0{}^2(\lambda_m a)-J_0{}^2(\lambda_m b))}$$

$$\times\left[2(J_0(\lambda_m b)Y_0(\lambda_m r)-J_0(\lambda_m r)Y_0(\lambda_m b))+4\lambda_m r\left(J_0(\lambda_m b)Y_0{}'(\lambda_m r)-J_0{}'(\lambda_m r)Y_0(\lambda_m b)\right)\right.$$

$$\left.+\lambda_m{}^2r^2(J_0(\lambda_m b)Y_0{}'(\lambda_m r)-J_0{}'(\lambda_m r)Y_0(\lambda_m b))\right]\times\int_0^t \overline{u}(n,t')e^{-k(\lambda_m{}^2+p^2)(t-t')}\,dt'$$

$$+\lambda\left(\tfrac{1+\nu}{1-\nu}\right)\frac{a_t}{4}\sum_{n=1}^{\infty}r^2p^2\cos(pz)\times\sum_{m=1}^{\infty}\frac{\lambda_m{}^2J_0{}^2(\lambda_m b)}{(\lambda_m{}^2+p^2)[J_0{}^2(\lambda_m a)-J_0{}^2(\lambda_m b)]}$$

$$\times[J_0(\lambda_m b)Y_0(\lambda_m r)-J_0(\lambda_m r)Y_0(\lambda_m b)]\times\int_0^t \overline{u}(n,t')e^{-k(\lambda_m{}^2+p^2)(t-t')}\,dt' \qquad (11.3)$$

$$\tau_{rz}=-G\left(\tfrac{1+\nu}{1-\nu}\right)\frac{a_t}{2}\sum_{n=1}^{\infty}p\sin(pz)\times\sum_{m=1}^{\infty}\frac{\lambda_m{}^2J_0(\lambda_m a)J_0(\lambda_m b)}{(\lambda_m{}^2+p^2)(J_0{}^2(\lambda_m a)-J_0{}^2(\lambda_m b))}$$

$$\times\left[2r(J_0(\lambda_m a)Y_0(\lambda_m r)-J_0(\lambda_m r)Y_0(\lambda_m a))+\lambda_m r^2\left(J_0(\lambda_m a)Y_0'(\lambda_m r)-J_0'(\lambda_m r)Y_0(\lambda_m a)\right)\right.$$

$$\times\int_0^t \bar{f}(n,t')e^{-k(\lambda_m^2+p^2)(t-t')}\,dt'$$

$$+G\left(\frac{1+\nu}{1-\nu}\right)\frac{a_t}{2}\sum_{n=1}^{\infty}p\sin(pz)\times\sum_{m=1}^{\infty}\frac{\lambda_m^2 J_0^2(\lambda_m b)}{(\lambda_m^2+p^2)(J_0^2(\lambda_m a)-J_0^2(\lambda_m b))}$$

$$\times\left[2r(J_0(\lambda_m b)Y_0(\lambda_m r)-J_0(\lambda_m r)Y_0(\lambda_m b))+\lambda_m r^2\left(J_0(\lambda_m b)Y_0'(\lambda_m r)-J_0'(\lambda_m r)Y_0(\lambda_m b)\right)\right.$$

$$\times\int_0^t \bar{u}(n,t')e^{-k(\lambda_m^2+p^2)(t-t')}\,dt' \qquad (11.4)$$

12. Special Case

Set $f(z,t)=(1-e^{-t})e^{-z^2}b$, $u(z,t)=(1-e^{-t})e^{-z^2}a$ (12.1)

Applying Fourier cosine transform to the equation (12.1) one obtains

$$\bar{f}(n,t)=\int_0^{\infty}(1-e^{-t})e^{-z^2}b\cos(pz)dz=\frac{(1-e^{-t})b\sqrt{\pi}}{2}\left[e^{-p^2/4}\right] \qquad (12.2)$$

$$\bar{u}(n,t)=\int_0^{\infty}(1-e^{-t})e^{-z^2}a\cos(pz)dz=\frac{(1-e^{-t})a\sqrt{\pi}}{2}\left[e^{-p^2/4}\right] \qquad (12.3)$$

Substituting the values of $\bar{f}(n,t)$ and $\bar{u}(n,t)$ from (12.2) and (12.3) in the equations (9.20) one obtains

$$T(r,z,t)=\frac{b\sqrt{\pi}}{2}\sum_{n=1}^{\infty}\left[e^{-p^2/4}\right]\cos(pz)\times\sum_{m=1}^{\infty}\frac{\lambda_m^2 J_0(\lambda_m a)J_0(\lambda_m b)}{(\lambda_m^2+p^2)(J_0^2(\lambda_m a)-J_0^2(\lambda_m b))}$$

$$\times[J_0(\lambda_m a)Y_0(\lambda_m r)-J_0(\lambda_m r)Y_0(\lambda_m a)]\times\int_0^t(1-e^{-t'})e^{-k(\lambda_m^2+p^2)(t-t')}\,dt'$$

$$-\frac{a\sqrt{\pi}}{2}\sum_{n=1}^{\infty}\left[e^{-p^2/4}\right]\cos(pz)\times\sum_{m=1}^{\infty}\frac{\lambda_m^2 J_0^2(\lambda_m b)}{(\lambda_m^2+p^2)(J_0^2(\lambda_m a)-J_0^2(\lambda_m b))}$$

$$\times[J_0(\lambda_m b)Y_0(\lambda_m r)-J_0(\lambda_m r)Y_0(\lambda_m b)]\times\int_0^t(1-e^{-t'})e^{-k(\lambda_m^2+p^2)(t-t')}\,dt' \qquad (12.4)$$

13. Numerical Results

Set $\alpha = \dfrac{\sqrt{\pi}}{2}$, a = 1 m, b= 2 m, t =1 sec, k = 0.86 and λ_m are the roots of the transcendental equation $J_0(\lambda_m a)Y_0(\lambda_m b) - J_0(\lambda_m b)Y_0(\lambda_m a) = 0$ as Ozisik [16] in (12.4) to obtain

$$\frac{T(r,z,t)}{\alpha} = \sum_{n=1}^{\infty}\left[e^{-p^2/4}\right]\cos(pz) \times \left\{ \sum_{m=1}^{\infty} \frac{\lambda_m{}^2 J_0(\lambda_m) J_0(2\lambda_m)(2)}{(\lambda_m{}^2 + p^2)(J_0{}^2(\lambda_m) - J_0{}^2(2\lambda_m))} \right.$$

$$\times [J_0(\lambda_m)Y_0(\lambda_m r) - J_0(\lambda_m r)Y_0(\lambda_m)]$$

$$- \sum_{m=1}^{\infty} \frac{\lambda_m{}^2 J_0{}^2(2\lambda_m)}{(\lambda_m{}^2 + p^2)(J_0{}^2(\lambda_m) - J_0{}^2(2\lambda_m))}$$

$$\left. \times [J_0(2\lambda_m)Y_0(\lambda_m r) - J_0(\lambda_m r)Y_0(2\lambda_m)] \right\} \times \int_0^1 (1 - e^{-t'}) e^{-0.86(\lambda_m{}^2 + p^2)(1-t')}\, dt' \qquad (13.1)$$

14. Conclusion

In this section, we derived the expressions of temperature distribution, displacement function and thermal stresses of semi-infinite hollow cylinder on outer curved surface, when the boundary conditions are known. Fourier cosine transform and Laplace transform techniques are used to obtain the numerical results. The solutions are obtained in terms of Bessel's function in the form of infinite series. The results that are obtained can be applied to the design of useful structures or machines in engineering applications. Any particular case of special interest can be derived by assigning suitable values to the parameters and functions in the expressions.

REFERENCES

[1] W. Nowacki, Thermoelasticity, Mass Addition-Wesley Publ. Co. 1962.

[2] Sierakowski and Sun, An exact solution to the elastic deformation of a finite length hollow cylinder, Journal of the Franklin Institute, 286 , pp.99-113, 1968

[3] D. B. Kamdi, N. W. Khobragade and M. H. Durge, Transient thermoelastic problem for a circular solid cylinder with radiation, Int. Journal of Pure and Applied Maths, vol. 54, No. 3, 387-406, 2009

[4] R. T. Walde and N. W. Khobragade, Transient thermoelastic problem of a finite length hollow cylinder, Canadian Journal on Science and Engg. Mathematics Research, Vol. 3 No. 2, pp. 56-60, 2012

[5] Ashwini A. Kulkarni and N. W. Khobragade, Thermal stresses of a finite length hollow cylinder, Canadian Journal on Science and Engg. Mathematics Research, Vol. 3 No. 2, pp. 52-55, 2012

[6] M. S. Warbhe and N.W. Khobragade, Numerical study of transient thermoelastic problem of a finite length hollow cylinder, Int. Journal of Latest Trends in Maths, Vol. 2, No. 1, pp. 4-9, 2012

[7] N. K. Lamba, and N. W. Khobragade, analysis of coupled thermal stresses in a axisymmetric hollow cylinder, Int. Journal of Latest Trends in Maths, Vol. 1, No. 2, 2011.

[8] Payal C. Hiranwar and N. W. Khobragade, Thermoelastic problem of a cylinder with internal heat sources, Journal of Statistics and Mathematics, Vol. 3, Issue 2, pp. 87-93, 2012

[9] S. D. Bagde and N.W Khobragade, Heat conduction problem for a finite elliptic cylinder, Int. Journal of Scientific & Engineering Research, vol. 3, Issue 8, pp. 1-3, 2012.

[10] N. W. Khobragade, Lalsingh Khalsa and Ashwini Kulkarni, Thermal deflection of a finite length hollow cylinder due to heat generation, Int. J. of Engg. And Innovative Technology, vol. 3, Issue 1, pp. 372-375, 2013

[11] N.W. Khobragade, Thermoelastic analysis of a thick hollow cylinder with radiation conditions, Int. J. of Engg. And Innovative Technology, vol. 3, Issue 4, pp.380-387, 2013

[12] Ranjana S. Ghume, Ashwini Mahakalkar, N. W. Khobragade, Interior Thermo elastic Solution of a Hollow Cylinder. IJEIT Volume 4, Issue 1, pp 206-211, 2014

[13] S. S. Singru, Sachin Chauthale and N. W. Khobragade, Thermal stress analysis of a thick hollow cylinder, IJEIT Volume 5, Issue 3, pp. 157-160, 2015

[14] S. S. Singru and N. W. Khobragade, Integral transform methods for inverse problem of heat conduction with known boundary of semi-infinite hollow cylinder and its stresses , IJLTEMAS Volume 6, Issue 4, pp 05-09. 2017

[15] Vivek Fule, M. S. Warbhe and N. W. Khobragade, Thermal stresses of semi-infinite hollow cylinder with internal heat source, , IJLTEMAS Volume 7, Issue 2, pp 29-34. 2018

[16] N. M. Ozisik, Boundary Value Problem of Heat Conduction, International Textbook Company Scranton, Pennsylvania, 1968